부록 E 해답

10장

연습문제 10.1

01. $(4, 0, -3)$ **02.** C; A
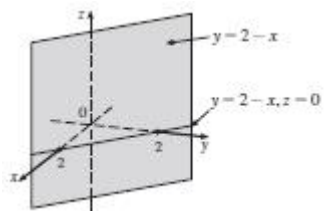
03. xy평면과 직선 $y = 2 - x$, $z = 0$에서 교차하는 수직평면;

04. (a) $|PQ| = 6$, $|QR| = 2\sqrt{10}$, $|RP| = 6$; 이등변삼각형

    (b) $|PQ| = 3$, $|QR| = 3\sqrt{5}$, $|RP| = 6$; 직각삼각형

05. (a) 아니다. (b) 직선 위에 있다.

06. $(x-3)^2 + (y-8)^2 + (z-1)^2 = 30$ **07.** $(1, 2, -4)$, 6

08. $(2, 0, -6)$, $9/\sqrt{2}$ **09.** (a) 생략   (b) $\dfrac{5}{2}$, $\dfrac{1}{2}\sqrt{94}$, $\dfrac{1}{2}\sqrt{85}$

10. (a) $(x-2)^2 + (y+3)^2 + (z-6)^2 = 36$ (b) $(x-2)^2 + (y+3)^2 + (z-6)^2 = 4$

    (c) $(x-2)^2 + (y+3)^2 + (z-6)^2 = 9$

11. yz평면 앞의 5 단위에 놓이고, 이 평면과 평행한 평면

12. 평면 $y = 8$ 왼쪽에 있는 모든 점들로 구성된 반공간

13. 수평평면 $z = 0$과 $z = 6$ 사이에 있는 모든 점들

14. 중심이 O이고 반지름이 $\sqrt{3}$인 구 위의 내부에 있는 모든 점들

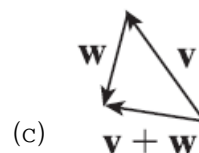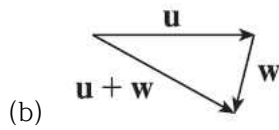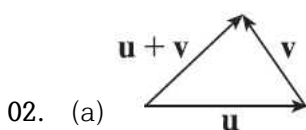**15.** 중심축이 y축이고 반지름이 3인 원기둥 위와 내부의 모든 점들

16. $0 < x < 5$ **17.** $r^2 < x^2 + y^2 + z^2 < R^2$

18. $14x - 6y - 10z = 9$, AB에 수직인 평면   **19.** $2\sqrt{3} - 3$

연습문제 10.2

01. $\overrightarrow{AB} = \overrightarrow{DC}$, $\overrightarrow{DA} = \overrightarrow{CB}$, $\overrightarrow{DE} = \overrightarrow{EB}$, $\overrightarrow{EA} = \overrightarrow{CE}$

02. (a)   (b)   (c)

(d) **u − v**   (e)   (f) **u − w − v**

03. $a = \langle 4,\ 1 \rangle$;
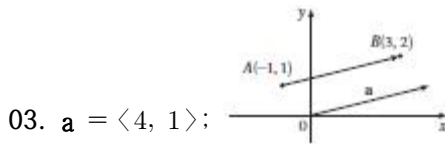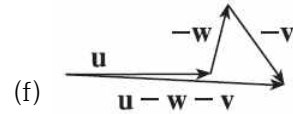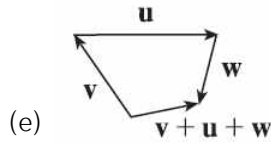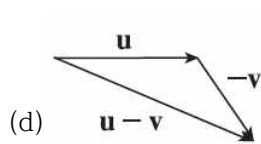

04. $a = \langle 2,\ 0,\ -2 \rangle$;

05. $\langle 5,\ 2 \rangle$;

06. $\langle 3,\ 8,\ 1 \rangle$;
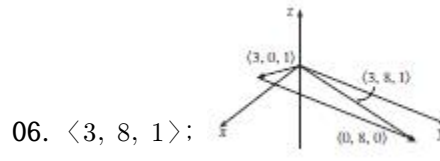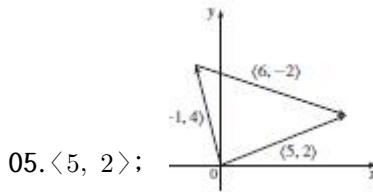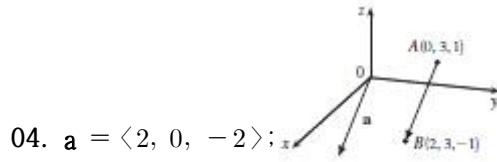
07. $\langle 2,\ -18 \rangle,\ \langle 1,\ -42 \rangle,\ 13,\ 10$

08. $-i + j + 2k,\ -4i + j + 9k,\ \sqrt{14},\ \sqrt{82}$

09. $\dfrac{8}{9}i - \dfrac{1}{9}j + \dfrac{4}{9}k$ 10. $60°$

11. $\langle 2,\ 2\sqrt{3} \rangle$ 12. $\approx 45.96\,\mathrm{ft/s},\ \approx 38.57\,\mathrm{ft/s}$

13. $100\sqrt{7} \approx 264.6\,\mathrm{N},\ \approx 139.1°$ 14. $\sqrt{1250} \approx 35.4\,\mathrm{km/h},\ \mathrm{N}8°\mathrm{W}$

15. $T_1 = -196i + 3.92j,\ T_2 = 196i + 3.92j$

16. (a) 제방에서 43.4°의 각으로 상류를 향하여   (b) 20.2분

17. $\pm(i + 4j)/\sqrt{17}$

18. (a), (b)   (c) $s \approx 1.3,\ t \approx 1.6$ (d) $s = \dfrac{9}{7},\ t = \dfrac{11}{7}$

19.
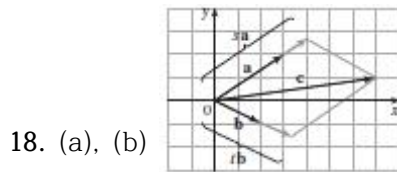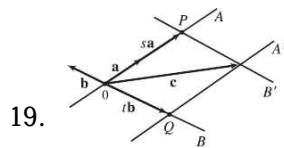
20. 중심이 $(x_0,\ y_0,\ z_0)$이고 반지름이 1인 구   21. 생략   22. 생략

연습문제 10.3

01. 의미가 있는 것은 (b), (c), (d)이다.   02. 14   03. 19   04. 1

05. -15　　　06. $\mathbf{u} \cdot \mathbf{v} = \dfrac{1}{2}$, $\mathbf{u} \cdot \mathbf{w} = -\dfrac{1}{2}$　　07. 생략　　08. $\cos^{-1}\left(\dfrac{1}{\sqrt{5}}\right) \approx 63°$

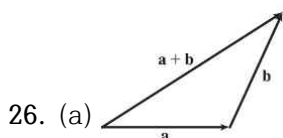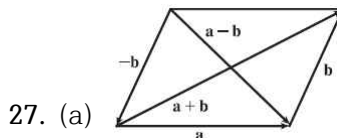09. $\cos^{-1}\left(\dfrac{7}{\sqrt{130}}\right) \approx 52°$　　10. (a) 아무것도 아니다.　(b) 수직　(c) 수직　(d) 평행

11. 그렇다.　　　　　　　　　　12. $(\mathbf{i} - \mathbf{j} - \mathbf{k})/\sqrt{3}$ 또는 $(-\mathbf{i} + \mathbf{j} + \mathbf{k})/\sqrt{3}$

13. $45°$　　　　　　　　　　14. $(0, 0)$에서 $0°$,　$(1, 1)$에서 $8.1°$

15. 4, $\left\langle -\dfrac{20}{13}, \dfrac{48}{13} \right\rangle$　　　　　16. $\dfrac{9}{7}$, $\left\langle \dfrac{27}{49}, \dfrac{54}{49}, -\dfrac{18}{49} \right\rangle$

17. $\mathrm{orth}_{\mathbf{a}}\mathbf{b} \cdot \mathbf{a} = (\mathbf{b} - \mathrm{proj}_{\mathbf{a}}\mathbf{b}) \cdot \mathbf{a} = \mathbf{b} \cdot \mathbf{a} - (\mathrm{proj}_{\mathbf{a}}\mathbf{b}) \cdot \mathbf{a} = \mathbf{b} \cdot \mathbf{a} - \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2}\mathbf{a} \cdot \mathbf{a}$

$$= \mathbf{b} \cdot \mathbf{a} - \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2}|\mathbf{a}|^2 = \mathbf{b} \cdot \mathbf{a} - \mathbf{a} \cdot \mathbf{b} = 0$$

18. $\langle 0, 0, -2\sqrt{10} \rangle$ 또는 $\langle s, t, 3s - 2\sqrt{10} \rangle$, $s, t \in \mathbb{R}$ 형태의 벡터

19. $144\,\mathrm{J}$　　　20. $560\cos 20° \approx 526\,\mathrm{J}$　　21. $\dfrac{13}{5}$　　　22. $\cos^{-1}(1/\sqrt{3}) \approx 55°$

23. 생략　　　　　　24. 생략　　　　　　25. 생략

26. (a) 　　(b) 생략　　27. (a) 　　(b) 생략

연습문제 10.4

01. $16\mathbf{i} + 48\mathbf{k}$　　　　　　02. $15\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$　　　　　03. $\dfrac{1}{2}\mathbf{i} - \mathbf{j} + \dfrac{3}{2}\mathbf{k}$

04. $(1-t)\mathbf{i} + (t^3 - t^2)\mathbf{k}$　　05. 0　　　　　　06. $\mathbf{i} + \mathbf{j} + \mathbf{k}$

07. (a) 스칼라　(b) 의미가 없다.　(c) 벡터　(d) 의미가 없다.
　　(e) 의미가 없다.　(f) 스칼라

08. $96\sqrt{3}$; 평면 안으로　　　　　　09. $\langle -7, 10, 8 \rangle$, $\langle 7, -10, -8 \rangle$

10. $\left\langle -\dfrac{1}{3\sqrt{3}}, -\dfrac{1}{3\sqrt{3}}, \dfrac{5}{3\sqrt{3}} \right\rangle$, $\left\langle \dfrac{1}{3\sqrt{3}}, \dfrac{1}{3\sqrt{3}}, -\dfrac{5}{3\sqrt{3}} \right\rangle$

11. $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$

$$\mathbf{0} \times \mathbf{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0 \\ a_1 & a_2 & a_3 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ a_2 & a_3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 0 & 0 \\ a_1 & a_3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 0 & 0 \\ a_1 & a_2 \end{vmatrix}\mathbf{k} = \mathbf{0},$$

$$\mathbf{a} \times \mathbf{0} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ 0 & 0 & 0 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ 0 & 0 \end{vmatrix}\mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ 0 & 0 \end{vmatrix}\mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ 0 & 0 \end{vmatrix}\mathbf{k} = \mathbf{0}.$$

12.
$$\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$$
$$= \langle (-1)(b_2 a_3 - b_3 a_2), (-1)(b_3 a_1 - b_1 a_3), (-1)(b_1 a_2 - b_2 a_1) \rangle$$
$$= -\langle b_2 a_3 - b_3 a_2, b_3 a_1 - b_1 a_3, b_1 a_2 - b_2 a_1 \rangle = -\mathbf{b} \times \mathbf{a}$$

13.
$$\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \langle b_1 + c_1, b_2 + c_2, b_3 + c_3 \rangle$$
$$= \langle a_2(b_3 + c_3) - a_3(b_2 + c_2), a_3(b_1 + c_1) - a_1(b_3 + c_3), a_1(b_2 + c_2) - a_2(b_1 + c_1) \rangle$$
$$= \langle a_2 b_3 + a_2 c_3 - a_3 b_2 - a_3 c_2, a_3 b_1 + a_3 c_1 - a_1 b_3 - a_1 c_3, a_1 b_2 + a_1 c_2 - a_2 b_1 - a_2 c_1 \rangle$$
$$= \langle (a_2 b_3 - a_3 b_2) + (a_2 c_3 - a_3 c_2), (a_3 b_1 - a_1 b_3) + (a_3 c_1 - a_1 c_3), (a_1 b_2 - a_2 b_1) + (a_1 c_2 - a_2 c_1) \rangle$$
$$= \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle + \langle a_2 c_3 - a_3 c_2, a_3 c_1 - a_1 c_3, a_1 c_2 - a_2 c_1 \rangle$$
$$= (\mathbf{a} \times \mathbf{b}) + (\mathbf{a} \times \mathbf{c})$$

14. 16   15. (a) $\langle 0, 18, -9 \rangle$ (b) $\dfrac{9}{2}\sqrt{5}$

16. (a) $\langle 13, -14, 5 \rangle$ (b) $\dfrac{1}{2}\sqrt{390}$

17. 9   18. 16

19.
$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 1 & 5 & -2 \\ 3 & -1 & 0 \\ 5 & 9 & -4 \end{vmatrix} = 1 \begin{vmatrix} -1 & 0 \\ 9 & -4 \end{vmatrix} - 5 \begin{vmatrix} 3 & 0 \\ 5 & -4 \end{vmatrix} + (-2) \begin{vmatrix} 3 & -1 \\ 5 & 9 \end{vmatrix} = 4 + 60 - 64 = 0.$$

20. $10.8 \sin 80° \approx 10.6 \, \text{N·m}$

21. $\approx 417 \, \text{N}$ 22. $60°$

23. (a)   (b) $\sqrt{97/3}$

24.
$$|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 \sin^2 \theta = |\mathbf{a}|^2 |\mathbf{b}|^2 (1 - \cos^2 \theta)$$
$$= |\mathbf{a}|^2 |\mathbf{b}|^2 - (|\mathbf{a}| |\mathbf{b}| \cos \theta)^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 - (\mathbf{a} \cdot \mathbf{b})^2$$

25.
$$(\mathbf{a} - \mathbf{b}) \times (\mathbf{a} + \mathbf{b}) = (\mathbf{a} - \mathbf{b}) \times \mathbf{a} + (\mathbf{a} - \mathbf{b}) \times \mathbf{b}$$
$$= \mathbf{a} \times \mathbf{a} + (-\mathbf{b}) \times \mathbf{a} + \mathbf{a} \times \mathbf{b} + (-\mathbf{b}) \times \mathbf{b}$$
$$= (\mathbf{a} \times \mathbf{a}) - (\mathbf{b} \times \mathbf{a}) + (\mathbf{a} \times \mathbf{b}) - (\mathbf{b} \times \mathbf{b})$$
$$= \mathbf{0} - (\mathbf{b} \times \mathbf{a}) + (\mathbf{a} \times \mathbf{b}) - \mathbf{0}$$
$$= (\mathbf{a} \times \mathbf{b}) + (\mathbf{a} \times \mathbf{b})$$
$$= 2(\mathbf{a} \times \mathbf{b})$$

26. $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$, $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, $\mathbf{c} = \langle c_1, c_2, c_3 \rangle$

$$\mathbf{b} \times \mathbf{c} = \langle b_2 c_3 - b_3 c_2, b_3 c_1 - b_1 c_3, b_1 c_2 - b_2 c_1 \rangle$$

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \langle a_2(b_1c_2 - b_2c_1) - a_3(b_3c_1 - b_1c_3), \ a_3(b_2c_3 - b_3c_2) - a_1(b_1c_2 - b_2c_1),$$
$$a_1(b_3c_1 - b_1c_3) - a_2(b_2c_3 - b_3c_2) \rangle$$

$$= \langle a_2b_1c_2 - a_2b_2c_1 - a_3b_3c_1 + a_3b_1c_3, \ a_3b_2c_3 - a_3b_3c_2 - a_1b_1c_2 + a_1b_2c_1,$$
$$a_1b_3c_1 - a_1b_1c_3 - a_2b_2c_3 + a_2b_3c_2 \rangle$$

$$= \langle (a_2c_2 + a_3c_3)b_1 - (a_2b_2 + a_3b_3)c_1, \ (a_1c_1 + a_3c_3)b_2 - (a_1b_1 + a_3b_3)c_2,$$
$$(a_1c_1 + a_2c_2)b_3 - (a_1b_1 + a_2b_2)c_3 \rangle$$

$$(\star) \quad = \langle (a_2c_2 + a_3c_3)b_1 - (a_2b_2 + a_3b_3)c_1 + a_1b_1c_1 - a_1b_1c_1,$$
$$(a_1c_1 + a_3c_3)b_2 - (a_1b_1 + a_3b_3)c_2 + a_2b_2c_2 - a_2b_2c_2,$$
$$(a_1c_1 + a_2c_2)b_3 - (a_1b_1 + a_2b_2)c_3 + a_3b_3c_3 - a_3b_3c_3 \rangle$$

$$= \langle (a_1c_1 + a_2c_2 + a_3c_3)b_1 - (a_1b_1 + a_2b_2 + a_3b_3)c_1,$$
$$(a_1c_1 + a_2c_2 + a_3c_3)b_2 - (a_1b_1 + a_2b_2 + a_3b_3)c_2,$$
$$(a_1c_1 + a_2c_2 + a_3c_3)b_3 - (a_1b_1 + a_2b_2 + a_3b_3)c_3 \rangle$$

$$= (a_1c_1 + a_2c_2 + a_3c_3)\langle b_1, b_2, b_3 \rangle - (a_1b_1 + a_2b_2 + a_3b_3)\langle c_1, c_2, c_3 \rangle$$

$$= (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b})$$

27.
$$= [(\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}] + [(\mathbf{b} \cdot \mathbf{a})\mathbf{c} - (\mathbf{b} \cdot \mathbf{c})\mathbf{a}] + [(\mathbf{c} \cdot \mathbf{b})\mathbf{a} - (\mathbf{c} \cdot \mathbf{a})\mathbf{b}]$$
$$= (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c} + (\mathbf{a} \cdot \mathbf{b})\mathbf{c} - (\mathbf{b} \cdot \mathbf{c})\mathbf{a} + (\mathbf{b} \cdot \mathbf{c})\mathbf{a} - (\mathbf{a} \cdot \mathbf{c})\mathbf{b} = \mathbf{0}$$

28. (a) 아니다.   (b) 아니다.   (c) 성립한다.

연습문제 10.5

01. (a) 참   (b) 거짓   (c) 참   (d) 거짓   (e) 거짓   (f) 참   (g) 거짓   (h) 참
(i) 참   (j) 거짓   (k) 참

02. $\mathbf{r} = (2\mathbf{i} + 2.4\mathbf{j} + 3.5\mathbf{k}) + t(3\mathbf{i} + 2\mathbf{j} - \mathbf{k}); \ x = 2 + 3t, \ y = 2.4 + 2t, \ z = 3.5 - t$

03. $\mathbf{r} = (\mathbf{i} + 6\mathbf{k}) + t(\mathbf{i} + 3\mathbf{j} + \mathbf{k}); \ x = 1 + t, \ y = 3t, \ z = 6 + t$

04. $x = 2 + 2t, \ y = 1 + \dfrac{1}{2}t, \ z = -3 - 4t; \ (x-2)/2 = 2y - 2 = (z+3)/(-4)$

05. $x = 1 + t, \ y = -1 + 2t, \ z = 1 + t; \ x - 1 = (y+1)/2 = z - 1$

06. 평행하다.

07. (a) $(x-1)/(-1) = (y+5)/2 = (z-6)/(-3)$

(b) $(-1, -1, 0), \ \left(-\dfrac{3}{2}, 0, -\dfrac{3}{2}\right), \ (0, -3, 3)$

08. $\mathbf{r}(t) = (2\mathbf{i} - \mathbf{j} + 4\mathbf{k}) + t(2\mathbf{i} + 7\mathbf{j} - 3\mathbf{k}), \ 0 \le t \le 1$

09. 꼬임   **10.** $(4, -1, -5)$ **11.** $x + 4y + z = 4$

12. $5x - y - z = 7$ **13.** $x + y + z = 2$ **14.** $33x + 10y + 4z = 190$

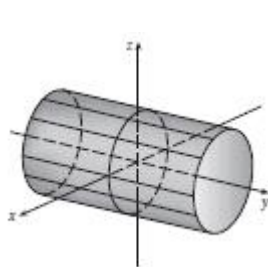**15.** $x - 2y + 4z = -1$ **16.** $3x - 8y - z = -38$ **17.** $(2, 3, 5)$
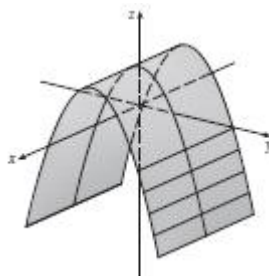
18. 어느 것도 아니다. $\cos^{-1}\left(\dfrac{1}{3}\right) \approx 70.5\,°$　　**19.** 평행

20. (a) $x = 1,\ y = -t,\ z = t$　　(b) $\cos^{-1}\left(\dfrac{5}{3\sqrt{3}}\right) \approx 15.8\,°$

21. $(x/a) + (y/b) + (z/c) = 1$　　　　　　**22.** $x = 3t,\ y = 1 - t,\ z = 2 - 2t$

23. P_2와 P_3이 평행, P_1과 P_4가 동일　　**24.** $\sqrt{61/14}$　　**25.** $\dfrac{18}{7}$

26. $5/(2\sqrt{14})$　　　　**27.** $D = \dfrac{|ax_0 + by_0 + cz_0 + d_2|}{\sqrt{a^2 + b^2 + c^2}} = \dfrac{|-d_1 + d_2|}{\sqrt{a^2 + b^2 + c^2}} = \dfrac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$
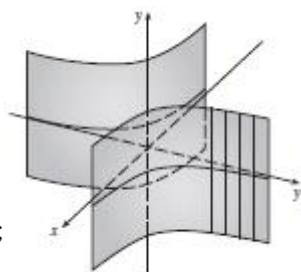
28. $1/\sqrt{6}$　　　　**29.** $13/\sqrt{69}$　　　　**30.** 생략

연습문제 10.6

01. (a) 포물선　　(b) z축에 평행하게 괘선을 그은 포물기둥

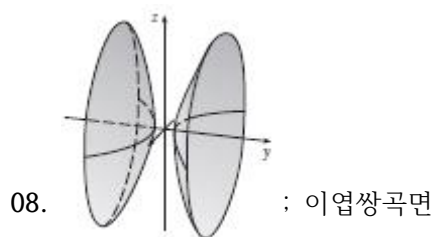　　(c) x축에 평행하게 괘선을 그은 포물기둥
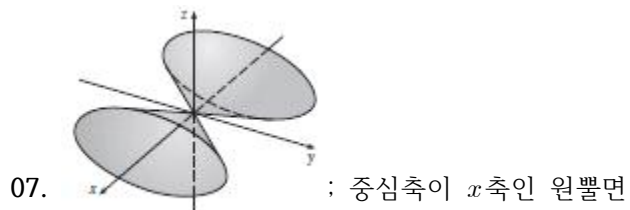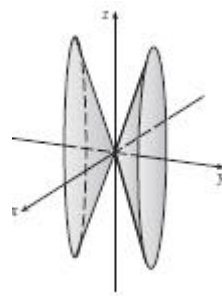

02. 원기둥;

03. 포물기둥;
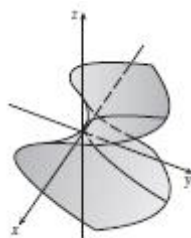

04. 쌍곡기둥;

05. (a) $x = k,\ y^2 - z^2 = 1 - k^2$, 쌍곡선 $(k \neq \pm 1)$;

　　　$y = k,\ x^2 - z^2 = 1 - k^2$, 쌍곡선 $(k \neq \pm 1)$;

　　　$z = k,\ x^2 + y^2 = 1 + k^2$, 원

　　(b) y축이 중심축이 되도록 쌍곡면이 회전된다.

　　(c) 쌍곡면이 음의 y방향으로 1단위만큼 이동된다.

06.   ; 중심축이 x축인 타원 포물면

07.   ; 중심축이 x축인 원뿔면

08.   ; 이엽쌍곡면
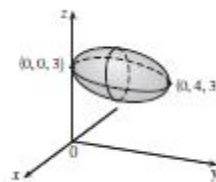
09. ; 타원면

10.   ; 쌍곡포물면
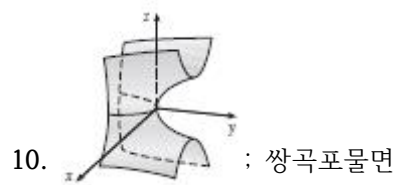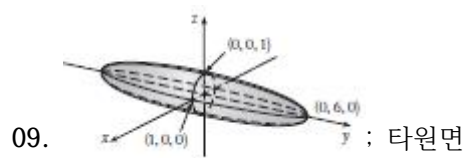
11. $y^2 = x^2 + \dfrac{z^2}{9}$, y축이 중심축인 원뿔면;
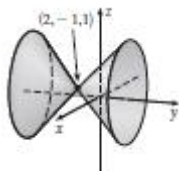

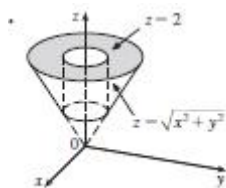12. $y = z^2 - \dfrac{x^2}{2}$, 쌍곡포물면;

13. $x^2 + \dfrac{(y-2)^2}{4} + (z-3)^2 = 1$, 중심이 $(0, 2, 3)$인 타원면;

14. $(y+1)^2 = (x-2)^2 + (z-1)^2$, 꼭짓점이 $(2, -1, 1)$, 중심축이 y축에 평행한 원뿔면;
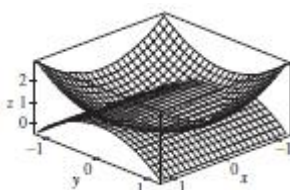

15.

16. $-4x = y^2 + z^2$, 포물면

17.

연습문제 10.7

01. $(-1, 2]$

02. $\mathbf{i} + \mathbf{j} + \mathbf{k}$

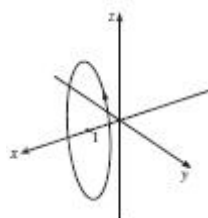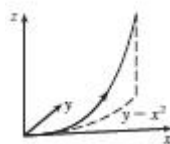
03.
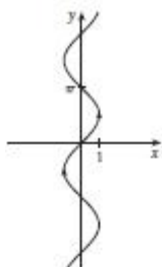
04.
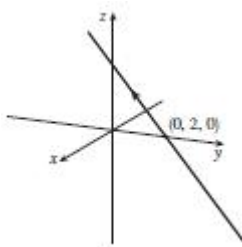
05.

06.

07. $\mathbf{r}(t) = \langle 2+4t,\ 2t,\ -2t \rangle,\ 0 \le t \le 1$; $x = 2+4t,\ y = 2t,\ z = -2t,\ 0 \le t \le 1$

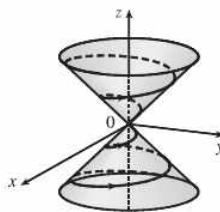08. $\mathbf{r}(t) = \left\langle \dfrac{1}{2}t,\ -1+\dfrac{4}{3}t,\ 1-\dfrac{3}{4}t \right\rangle,\ 0 \le t \le 1$;

$x = \dfrac{1}{2}t,\ y = -1+\dfrac{4}{3}t,\ z = 1-\dfrac{3}{4}t,\ 0 \le t \le 1$

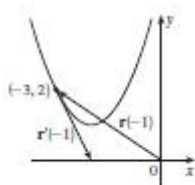**09.** II   **10.** V   **11.** IV   **12.**

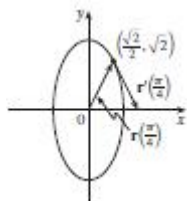**13.** $(0, 0, 0),\ (1, 0, 1)$ **14.** 생략

15. $\mathbf{r}(t) = t\,\mathbf{i} + \dfrac{1}{2}(t^2 - 1)\mathbf{j} + \dfrac{1}{2}(t^2 + 1)\mathbf{k}$
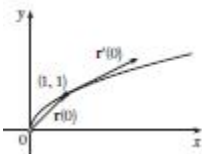
16. $x = 2\cos t,\ y = 2\sin t,\ z = 4\cos^2 t$

17. (a), (c)   (b) $\mathbf{r}'(t) = \langle 1,\ 2t \rangle$

18. (a), (c)   (b) $\mathbf{r}'(t) = \cos t\,\mathbf{i} - 2\sin t\,\mathbf{j}$

19. (a), (c)   (b) $\mathbf{r}'(t) = 2e^{2t}\mathbf{i} + e^t\mathbf{j}$

20. $\mathbf{r}'(t) = \langle t\cos t + \sin t,\ 2t,\ \cos 2t - 2t\sin 2t \rangle$

21. $\mathbf{r}'(t) = 2t\,e^{t^2}\mathbf{i} + [3/(1 + 3t)]\,\mathbf{k}$

22. $\mathbf{r}'(t) = \mathbf{b} + 2t\,\mathbf{c}$ **23.** $\dfrac{3}{5}\mathbf{j} + \dfrac{4}{5}\mathbf{k}$

24. $\langle 1,\ 2t,\ 3t^2 \rangle,\ \langle 1/\sqrt{14},\ 2/\sqrt{14},\ 3/\sqrt{14} \rangle,\ \langle 0,\ 2,\ 6t \rangle,\ \langle 6t^2,\ -6t,\ 2 \rangle$

25. $x = 3 + t,\ y = 2t,\ z = 2 + 4t$ **26.** $x = 1 - t,\ y = t,\ z = 1 - t$

27. $\mathbf{r}(t) = (3 - 4t)\mathbf{i} + (4 + 3t)\mathbf{j} + (2 - 6t)\mathbf{k}$ **28.** $x = -\pi - t,\ y = \pi + t,\ z = -\pi t$

29. $66\degree$ **30.** $2\mathbf{i} - 4\mathbf{j} + 32\mathbf{k}$ **31.** $\mathbf{i} + \mathbf{j} + \mathbf{k}$

32. $\tan t\,\mathbf{i} + \dfrac{1}{8}(t^2 + 1)^4\mathbf{j} + \left(\dfrac{1}{3}t^3\ln t - \dfrac{1}{9}t^3\right)\mathbf{k} + \mathbf{C}$

33. $t^2\mathbf{i} + t^3\mathbf{j} + \left(\dfrac{2}{3}t^{3/2} - \dfrac{2}{3}\right)\mathbf{k}$ **34.** 그렇다.

$$\lim_{t \to a} \mathbf{u}(t) + \lim_{t \to a} \mathbf{v}(t) = \left\langle \lim_{t \to a} u_1(t) + \lim_{t \to a} v_1(t), \lim_{t \to a} u_2(t) + \lim_{t \to a} v_2(t), \lim_{t \to a} u_3(t) + \lim_{t \to a} v_3(t) \right\rangle$$

$$= \left\langle \lim_{t \to a} [u_1(t) + v_1(t)], \lim_{t \to a} [u_2(t) + v_2(t)], \lim_{t \to a} [u_3(t) + v_3(t)] \right\rangle$$

$$= \lim_{t \to a} \left\langle u_1(t) + v_1(t), u_2(t) + v_2(t), u_3(t) + v_3(t) \right\rangle$$

35. (a)
$$= \lim_{t \to a} [\mathbf{u}(t) + \mathbf{v}(t)]$$

$$\lim_{t \to a} c\mathbf{u}(t) = \lim_{t \to a} \left\langle cu_1(t), cu_2(t), cu_3(t) \right\rangle = \left\langle \lim_{t \to a} cu_1(t), \lim_{t \to a} cu_2(t), \lim_{t \to a} cu_3(t) \right\rangle$$

$$= \left\langle c \lim_{t \to a} u_1(t), c \lim_{t \to a} u_2(t), c \lim_{t \to a} u_3(t) \right\rangle = c \left\langle \lim_{t \to a} u_1(t), \lim_{t \to a} u_2(t), \lim_{t \to a} u_3(t) \right\rangle$$

(b)
$$= c \lim_{t \to a} \left\langle u_1(t), u_2(t), u_3(t) \right\rangle = c \lim_{t \to a} \mathbf{u}(t)$$

$$\lim_{t \to a} \mathbf{u}(t) \cdot \lim_{t \to a} \mathbf{v}(t) = \left\langle \lim_{t \to a} u_1(t), \lim_{t \to a} u_2(t), \lim_{t \to a} u_3(t) \right\rangle \cdot \left\langle \lim_{t \to a} v_1(t), \lim_{t \to a} v_2(t), \lim_{t \to a} v_3(t) \right\rangle$$

$$= \left[\lim_{t \to a} u_1(t) \right] \left[\lim_{t \to a} v_1(t) \right] + \left[\lim_{t \to a} u_2(t) \right] \left[\lim_{t \to a} v_2(t) \right] + \left[\lim_{t \to a} u_3(t) \right] \left[\lim_{t \to a} v_3(t) \right]$$

$$= \lim_{t \to a} u_1(t)v_1(t) + \lim_{t \to a} u_2(t)v_2(t) + \lim_{t \to a} u_3(t)v_3(t)$$

(c)
$$= \lim_{t \to a} [u_1(t)v_1(t) + u_2(t)v_2(t) + u_3(t)v_3(t)] = \lim_{t \to a} [\mathbf{u}(t) \cdot \mathbf{v}(t)]$$

$$\lim_{t \to a} \mathbf{u}(t) \times \lim_{t \to a} \mathbf{v}(t) = \left\langle \lim_{t \to a} u_1(t), \lim_{t \to a} u_2(t), \lim_{t \to a} u_3(t) \right\rangle \times \left\langle \lim_{t \to a} v_1(t), \lim_{t \to a} v_2(t), \lim_{t \to a} v_3(t) \right\rangle$$

$$= \left\langle \left[\lim_{t \to a} u_2(t) \right] \left[\lim_{t \to a} v_3(t) \right] - \left[\lim_{t \to a} u_3(t) \right] \left[\lim_{t \to a} v_2(t) \right], \right.$$
$$\left[\lim_{t \to a} u_3(t) \right] \left[\lim_{t \to a} v_1(t) \right] - \left[\lim_{t \to a} u_1(t) \right] \left[\lim_{t \to a} v_3(t) \right],$$
$$\left. \left[\lim_{t \to a} u_1(t) \right] \left[\lim_{t \to a} v_2(t) \right] - \left[\lim_{t \to a} u_2(t) \right] \left[\lim_{t \to a} v_1(t) \right] \right\rangle$$

$$= \left\langle \lim_{t \to a} [u_2(t)v_3(t) - u_3(t)v_2(t)], \lim_{t \to a} [u_3(t)v_1(t) - u_1(t)v_3(t)], \right.$$
$$\left. \lim_{t \to a} [u_1(t)v_2(t) - u_2(t)v_1(t)] \right\rangle$$

(d)
$$= \lim_{t \to a} \left\langle u_2(t)v_3(t) - u_3(t)v_2(t), u_3(t)v_1(t) - u_1(t)v_3(t), u_1(t)v_2(t) - u_2(t)v_1(t) \right\rangle$$

$$= \lim_{t \to a} [\mathbf{u}(t) \times \mathbf{v}(t)]$$

36. 생략

$$\frac{d}{dt}[\mathbf{u}(t) + \mathbf{v}(t)] = \frac{d}{dt} \left\langle u_1(t) + v_1(t), u_2(t) + v_2(t), u_3(t) + v_3(t) \right\rangle$$

$$= \left\langle \frac{d}{dt}[u_1(t) + v_1(t)], \frac{d}{dt}[u_2(t) + v_2(t)], \frac{d}{dt}[u_3(t) + v_3(t)] \right\rangle$$

$$= \left\langle u_1'(t) + v_1'(t), u_2'(t) + v_2'(t), u_3'(t) + v_3'(t) \right\rangle$$

37.
$$= \left\langle u_1'(t), u_2'(t), u_3'(t) \right\rangle + \left\langle v_1'(t), v_2'(t), v_3'(t) \right\rangle = \mathbf{u}'(t) + \mathbf{v}'(t)$$

$$\frac{d}{dt}\left[\mathbf{u}(t) \times \mathbf{v}(t)\right] = \frac{d}{dt}\langle u_2(t)v_3(t) - u_3(t)v_2(t),\, u_3(t)v_1(t) - u_1(t)v_3(t),\, u_1(t)v_2(t) - u_2(t)v_1(t)\rangle$$

$$= \langle u_2'v_3(t) + u_2(t)v_3'(t) - u_3'(t)v_2(t) - u_3(t)v_2'(t),$$

$$u_3'(t)v_1(t) + u_3(t)v_1'(t) - u_1'(t)v_3(t) - u_1(t)v_3'(t),$$

$$u_1'(t)v_2(t) + u_1(t)v_2'(t) - u_2'(t)v_1(t) - u_2(t)v_1'(t)\rangle$$

$$= \langle u_2'(t)v_3(t) - u_3'(t)v_2(t),\, u_3'(t)v_1(t) - u_1'(t)v_3(t),\, u_1'(t)v_2(t) - u_2'(t)v_1(t)\rangle$$

$$+ \langle u_2(t)v_3'(t) - u_3(t)v_2'(t),\, u_3(t)v_1'(t) - u_1(t)v_3'(t),\, u_1(t)v_2'(t) - u_2(t)v_1'(t)\rangle$$

$$= \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t)$$

38.

39. $2t\cos t + 2\sin t - 2\cos t \sin t$　　　　**40.** 35

41. 생략　　　　　　　　**42.** 생략　　　　　　　　**43.** 생략

연습문제 10.8

01. $10\sqrt{10}$　　　　**02.** $\dfrac{1}{27}(13^{3/2} - 8)$　　　**03.** 18.6833　　　　**04.** 42

05. $\mathbf{r}(t(s)) = \dfrac{2}{\sqrt{29}}s\,\mathbf{i} + \left(1 - \dfrac{3}{\sqrt{29}}s\right)\mathbf{j} + \left(5 + \dfrac{4}{\sqrt{29}}s\right)\mathbf{k}$

06. $(3\sin 1,\, 4,\, 3\cos 1)$

07. (a) $\left\langle \dfrac{1}{\sqrt{10}},\, \dfrac{-3}{\sqrt{10}}\sin t,\, \dfrac{3}{\sqrt{10}}\cos t \right\rangle,\ \langle 0,\, -\cos t,\, -\sin t\rangle$　　　　(b) $\dfrac{3}{10}$

08. (a) $\dfrac{1}{e^{2t}+1}\langle \sqrt{2}\,e^t,\, e^{2t},\, -1\rangle,\ \dfrac{1}{e^{2t}+1}\langle 1-e^{2t},\, \sqrt{2}\,e^t,\, \sqrt{2}\,e^t\rangle$

　　　(b) $\sqrt{2}\,e^{2t}/(e^{2t}+1)^2$
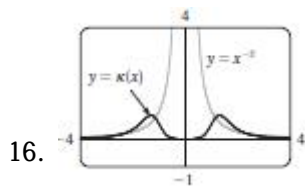
09. $6t^2/(9t^4+4t^2)^{3/2}$　　　**10.** $\dfrac{4}{25}$　　　**11.** $\dfrac{1}{7}\sqrt{\dfrac{19}{14}}$　　　**12.** $12x^2/(1+16x^6)^{3/2}$
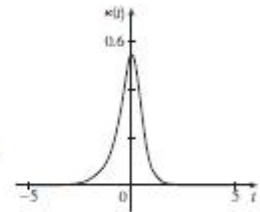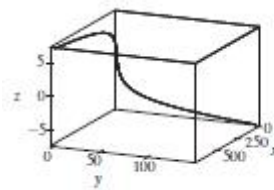
13. $e^x|x+2|/[1+(xe^x+e^x)^2]^{3/2}$　　　**14.** $\left(-\dfrac{1}{2}\ln 2,\, \dfrac{1}{\sqrt{2}}\right)$; 0으로 접근

15. (a) P　　(b) 1.3, 0.7

16. 　　　　　　**17.**

18. a는 $y = f(x)$이고, b는 $y = \kappa(x)$이다.

19. $\mathbf{r}(t) = \langle f(t), g(t)\rangle,\ \mathbf{r}'(t) = \langle f'(t), g'(t)\rangle,\ \mathbf{r}''(t) = \langle f''(t), g''(t)\rangle.$

$|\mathbf{r}'(t)|^3 = \left[\sqrt{(f'(t))^2 + (g'(t))^2}\right]^3 = [(f'(t))^2 + (g'(t))^2]^{3/2} = (\dot{x}^2 + \dot{y}^2)^{3/2},$

$|\mathbf{r}'(t) \times \mathbf{r}''(t)| = |\langle 0, 0, f'(t)g''(t) - f''(t)g'(t)\rangle| = [(\dot{x}\ddot{y} - \ddot{x}\dot{y})^2]^{1/2} = |\dot{x}\ddot{y} - \dot{y}\ddot{x}|.$

따라서 $\kappa(t) = \dfrac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{[\dot{x}^2 + \dot{y}^2]^{3/2}}$ 이다.
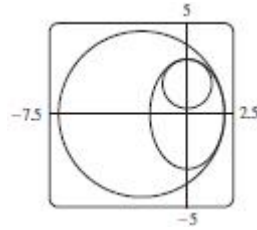
20. $1/(\sqrt{2}\, e^t)$

21. $x = 1 + t^3 \ \Rightarrow \ \dot{x} = 3t^2 \ \Rightarrow \ \ddot{x} = 6t, \ \ y = t + t^2 \ \Rightarrow \ \dot{y} = 1 + 2t \ \Rightarrow \ \ddot{y} = 2.$

$$\kappa(t) = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{[\dot{x}^2 + \dot{y}^2]^{3/2}} = \frac{\left|(3t^2)(2) - (1+2t)(6t)\right|}{[(3t^2)^2 + (1+2t)^2]^{3/2}} = \frac{|-6t^2 - 6t|}{(9t^4 + 4t^2 + 4t + 1)^{3/2}} = \frac{6\,|t^2 + t|}{(9t^4 + 4t^2 + 4t + 1)^{3/2}}.$$

22. $\left\langle \dfrac{2}{3}, \dfrac{2}{3}, \dfrac{1}{3} \right\rangle, \ \left\langle -\dfrac{1}{3}, \dfrac{2}{3}, -\dfrac{2}{3} \right\rangle, \ \left\langle -\dfrac{2}{3}, \dfrac{1}{3}, \dfrac{2}{3} \right\rangle$

23. $y = 6x + \pi, \ x + 6y = 6\pi$

24. $\left(x + \dfrac{5}{2}\right)^2 + y^2 = \dfrac{81}{4}, \ x^2 + \left(y - \dfrac{5}{3}\right)^2 = \dfrac{16}{9};$

25. $(-1, -3, 1)$ 　　　　　　 26. $2.07 \times 10^{10}\ \text{Å} \ \approx \ 2\,\text{m}$

27. (a) $|\mathbf{B}| = 1 \ \Rightarrow \ \mathbf{B} \cdot \mathbf{B} = 1 \ \Rightarrow \ \dfrac{d}{ds}(\mathbf{B} \cdot \mathbf{B}) = 0 \ \Rightarrow \ 2\dfrac{d\mathbf{B}}{ds} \cdot \mathbf{B} = 0 \ \Rightarrow \ \dfrac{d\mathbf{B}}{ds} \perp \mathbf{B}$

(b) $\mathbf{B} = \mathbf{T} \times \mathbf{N} \ \Rightarrow$

$$\frac{d\mathbf{B}}{ds} = \frac{d}{ds}(\mathbf{T} \times \mathbf{N}) = \frac{d}{dt}(\mathbf{T} \times \mathbf{N})\frac{1}{ds/dt} = \frac{d}{dt}(\mathbf{T} \times \mathbf{N})\frac{1}{|\mathbf{r}'(t)|} = [(\mathbf{T}' \times \mathbf{N}) + (\mathbf{T} \times \mathbf{N}')]\frac{1}{|\mathbf{r}'(t)|}$$

$$= \left[\left(\mathbf{T}' \times \frac{\mathbf{T}'}{|\mathbf{T}'|}\right) + (\mathbf{T} \times \mathbf{N}')\right]\frac{1}{|\mathbf{r}'(t)|} = \frac{\mathbf{T} \times \mathbf{N}'}{|\mathbf{r}'(t)|} \ \Rightarrow \ \frac{d\mathbf{B}}{ds} \perp \mathbf{T}$$

(c) 생략　　　　(d) 생략

28. (a) $\mathbf{r}' = s'\,\mathbf{T} \ \Rightarrow \ \mathbf{r}'' = s''\,\mathbf{T} + s'\,\mathbf{T}' = s''\,\mathbf{T} + s'\dfrac{d\mathbf{T}}{ds}s' = s''\,\mathbf{T} + \kappa(s')^2\,\mathbf{N}$

(b) $\mathbf{r}' \times \mathbf{r}'' = (s'\,\mathbf{T}) \times [s''\,\mathbf{T} + \kappa(s')^2\,\mathbf{N}]$

$$= [(s'\,\mathbf{T}) \times (s''\,\mathbf{T})] + [(s'\mathbf{T}) \times (\kappa(s')^2\,\mathbf{N})]$$

$$= (s's'')(\mathbf{T} \times \mathbf{T}) + \kappa(s')^3(\mathbf{T} \times \mathbf{N}) = \mathbf{0} + \kappa(s')^3\,\mathbf{B} = \kappa(s')^3\,\mathbf{B}$$

(c) $\mathbf{r}''' = [s''\,\mathbf{T} + \kappa(s')^2\,\mathbf{N}]' = s'''\,\mathbf{T} + s''\,\mathbf{T}' + \kappa'(s')^2\,\mathbf{N} + 2\kappa s's''\,\mathbf{N} + \kappa(s')^2\,\mathbf{N}'$

$$= s'''\,\mathbf{T} + s''\frac{d\mathbf{T}}{ds}s' + \kappa'(s')^2\,\mathbf{N} + 2\kappa s's''\,\mathbf{N} + \kappa(s')^2\frac{d\mathbf{N}}{ds}s'$$

$$= s'''\,\mathbf{T} + s''s'\kappa\,\mathbf{N} + \kappa'(s')^2\,\mathbf{N} + 2\kappa s's''\,\mathbf{N} + \kappa(s')^3(-\kappa\,\mathbf{T} + \tau\,\mathbf{B})$$

$$= [s''' - \kappa^2(s')^3]\,\mathbf{T} + [3\kappa s's'' + \kappa'(s')^2]\,\mathbf{N} + \kappa\tau(s')^3\,\mathbf{B}$$
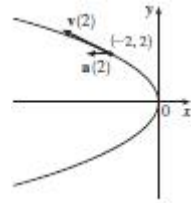
(d) $\mathbf{B} \cdot \mathbf{T} = 0, \ \mathbf{B} \cdot \mathbf{N} = 0, \ \mathbf{B} \cdot \mathbf{B} = 1$

$$\frac{(\mathbf{r}' \times \mathbf{r}'') \cdot \mathbf{r}'''}{|\mathbf{r}' \times \mathbf{r}''|^2} = \frac{\kappa(s')^3 \mathbf{B} \cdot \{[s''' - \kappa^2(s')^3]\mathbf{T} + [3\kappa s' s'' + \kappa'(s')^2]\mathbf{N} + \kappa\tau(s')^3\mathbf{B}\}}{|\kappa(s')^3 \mathbf{B}|^2}$$

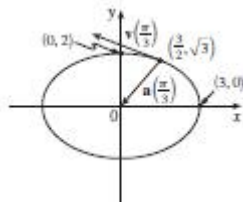$$= \frac{\kappa(s')^3 \kappa\tau(s')^3}{[\kappa(s')^3]^2} = \tau.$$

29. $L = \int_0^{2.9\times10^8\times2\pi} |\mathbf{r}'(t)|\, dt = \int_0^{2.9\times10^8\times2\pi} \sqrt{(-10\sin t)^2 + (10\cos t)^2 + \left(\frac{34}{2\pi}\right)^2}\, dt$

$$= \sqrt{100 + \left(\tfrac{34}{2\pi}\right)^2}\, t \,\Big]_0^{2.9\times10^8\times2\pi}$$

$$= 2.9\times10^8 \times 2\pi \sqrt{100 + \left(\tfrac{34}{2\pi}\right)^2} \approx 2.07 \times 10^{10} \text{ Å}\cdot$$
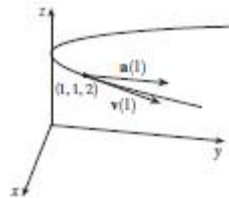
연습문제 10.9

01. $\mathbf{v}(t) = \langle -t,\, 1\rangle$, $\mathbf{a}(t) = \langle -1,\, 0\rangle$, $|\mathbf{v}(t)| = \sqrt{t^2+1}$;

02. $\mathbf{v}(t) = -3\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j}$, $\mathbf{a}(t) = -3\cos t\,\mathbf{i} - 2\sin t\,\mathbf{j}$, $|\mathbf{v}(t)| = \sqrt{5\sin^2 t + 4}$;

03. $\mathbf{v}(t) = \mathbf{i} + 2t\,\mathbf{j}$, $\mathbf{a}(t) = 2\mathbf{j}$, $|\mathbf{v}(t)| = \sqrt{1 + 4t^2}$;
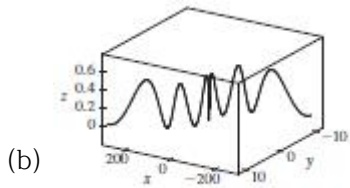
04. $\langle 2t,\, 3t^2,\, 2t\rangle$, $\langle 2,\, 6t,\, 2\rangle$, $|t|\sqrt{9t^2 + 8}$

05. $\sqrt{2}\,\mathbf{i} + e^t\mathbf{j} - e^{-t}\mathbf{k}$, $e^t\mathbf{j} + e^{-t}\mathbf{k}$, $e^t + e^{-t}$

06. $\mathbf{v}(t) = t\,\mathbf{i} + 2t\,\mathbf{j} + \mathbf{k}$, $\mathbf{r}(t) = \left(\frac{1}{2}t^2 + 1\right)\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}$

07. (a) $\mathbf{r}(t) = \left(\frac{1}{3}t^3 + t\right)\mathbf{i} + (t - \sin t + 1)\mathbf{j} + \left(\frac{1}{4} - \frac{1}{4}\cos 2t\right)\mathbf{k}$

(b)

08. $t = 4$

09. $\mathbf{r}(t) = t\mathbf{i} - t\mathbf{j} + \frac{5}{2}t^2\mathbf{k}$, $|\mathbf{v}(t)| = \sqrt{25t^2 + 2}$

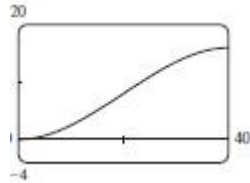**10.** (a) $\approx 22\,\mathrm{km}$ (b) $\approx 3.2\,\mathrm{km}$ (c) $500\,\mathrm{m/s}$

11. (a) $\approx 22.4\,\mathrm{km}$ (b) $\approx 3.4\,\mathrm{km}$ (c) $504\,\mathrm{m/s}$

12. $30\,\mathrm{m/s}$

13. $\approx 10.2°$, $\approx 79.8°$

14. $13.0° < \theta < 36.0°$, $55.4° < \theta < 85.5°$

15. $(250, -50, 0)$; $10\sqrt{93} \approx 96.4\,\mathrm{ft/s}$

16. (a) $16\,\mathrm{m}$;

(b) 상류로 $\approx 23.6°$;
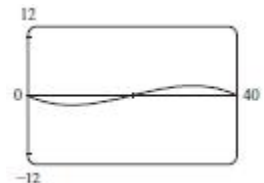
17. $0, 1$

18. $6t, 6$

19. $t = 1$

10장 복습문제

참-거짓 질문

01. 거짓   **02.** 거짓   **03.** 참   **04.** 참   **05.** 참   **06.** 참   **07.** 참

08. 거짓   **09.** 거짓   **10.** 거짓   **11.** 참   **12.** 참   **13.** 거짓   **14.** 거짓

15. 거짓   **16.** 참   **17.** 거짓

연습문제

01. (a) $(x+1)^2 + (y-2)^2 + (z-1)^2 = 69$ (b) $(y-2)^2 + (z-1)^2 = 68$, $x = 0$
(c) 중심 $(4, -1, -3)$, 반지름 5

02. $\mathbf{u} \cdot \mathbf{v} = 3\sqrt{2}$; $|\mathbf{u} \times \mathbf{v}| = 3\sqrt{2}$; 평면 밖으로   **03.** $-2, -4$

04. (a) 2 (b) -2 (c) -2 (d) 0

05. $\cos^{-1}\left(\frac{1}{3}\right) \approx 71°$

06. (a) $\langle 4, -3, 4 \rangle$ (b) $\sqrt{41}/2$

07. $166\,\mathrm{N}, 114\,\mathrm{N}$

08. $x = 4 - 3t$, $y = -1 + 2t$, $z = 2 + 3t$

09. $x = -2 + 2t$, $y = 2 - t$, $z = 4 + 5t$

10. $-4x + 3y + z = -14$

11. $(1, 4, 4)$
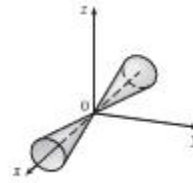
12. 꼬임

13. $x + y + z = 4$

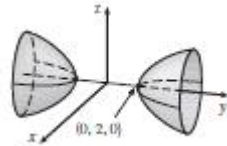**14.** $22/\sqrt{26}$
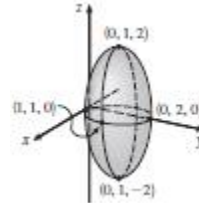
15. 평면;
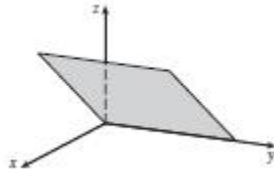
16. 원뿔면;
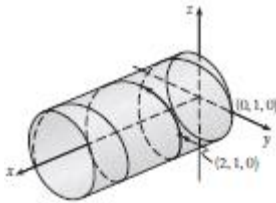
17. 이엽쌍곡면;

18. 타원면;

19. $4x^2 + y^2 + z^2 = 16$

20. (a)

(b) $\mathbf{r}'(t) = \mathbf{i} - \pi \sin \pi t\, \mathbf{j} + \pi \cos \pi t\, \mathbf{k}$, $\mathbf{r}''(t) = -\pi^2 \cos \pi t\, \mathbf{j} - \pi^2 \sin \pi t\, \mathbf{k}$

21. $\mathbf{r}(t) = 4\cos t\, \mathbf{i} + 4 \sin t\, \mathbf{j} + (5 - 4\cos t)\mathbf{k}$, $0 \le t \le 2\pi$

22. $\dfrac{1}{3}\mathbf{i} - (2/\pi^2)\mathbf{j} + (2/\pi)\mathbf{k}$ **23.** 86.631   **24.** $\pi/2$

25. (a) $\langle t^2,\ t,\ 1 \rangle / \sqrt{t^4 + t^2 + 1}$

(b) $\langle t^3 + 2t,\ 1 - t^4,\ -2t^3 - t \rangle / \sqrt{t^8 + 5t^6 + 6t^4 + 5t^2 + 1}$

(c) $\sqrt{t^8 + 5t^6 + 6t^4 + 5t^2 + 1} / (t^4 + t^2 + 1)^2$

26. $12/17^{3/2}$

27. $\mathbf{v}(t) = (1 + \ln t)\mathbf{i} + \mathbf{j} - e^{-t}\mathbf{k}$, $|\mathbf{v}(t)| = \sqrt{2 + 2\ln t + (\ln t)^2 + e^{-2t}}$,

$\mathbf{a}(t) = (1/t)\mathbf{i} + e^{-t}\mathbf{k}$

28. (a) 땅 위로 약 0.8 m, 선수로부터 18.4 m (b) ≈ 6.3 m (c) 선수로부터 ≈ 19.1 m

29. $\pi |t|$