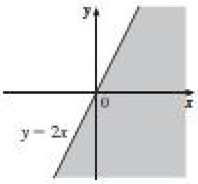
부록 E 해답

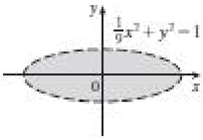
11장

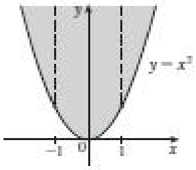
연습문제 11.1

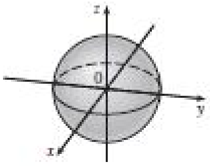
01. (a) 1 (b) $\mathbb{R}^2$ (c) $[-1, 1]$

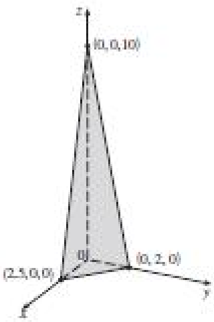
02. (a) 3 (b) $\left\{(x, y, z) \mid x^2+y^2+z^2 < 4,\ x \geq 0,\ y \geq 0,\ z \geq 0\right\}$
반지름이 2이고 중심이 원점인 제1팔분공간 안에 놓이는 구의 내부

03. $\{(x, y) \mid y \leq 2x\}$; 

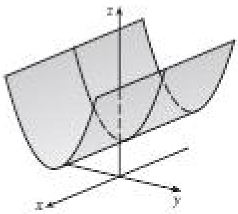
04. $\left\{(x, y) \mid \frac{1}{9}x^2+y^2 < 1\right\}$; 

05. $\left\{(x, y) \mid y \geq x^2,\ x \neq \pm 1\right\}$; 

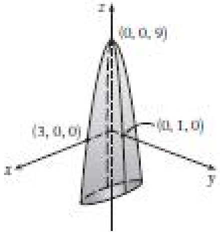
06. $\left\{(x, y, z) \mid x^2+y^2+z^2 \leq 1\right\}$; 

07. $4x+5y+z=10$, 평면 

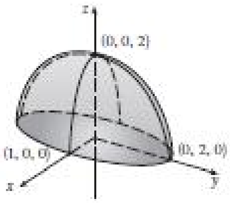
08. $z = y^2 + 1$, 포물기둥



09. $z = 9 - x^2 - 9y^2$, 타원포물면



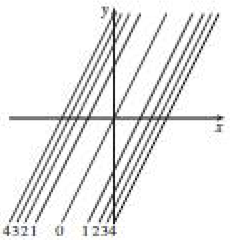
10. $z = \sqrt{4 - 4x^2 - y^2}$, 타원면의 상반부



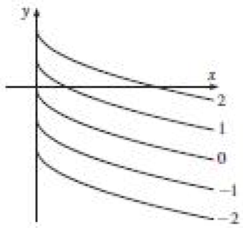
11. ≈ 56, ≈ 35

12. 가파른; 거의 평평한

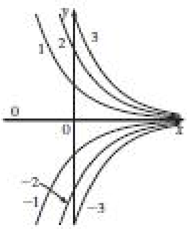
13. $(y - 2x)^2 = k$;



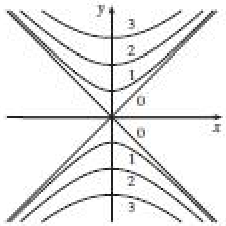
14. $y = -\sqrt{x} + k$;

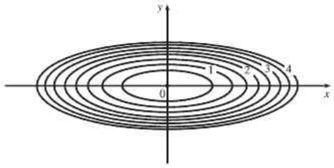


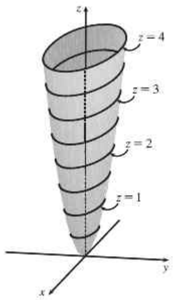
15. $y = ke^{-x}$;

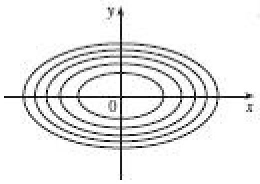


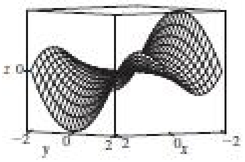
16. $y^2 - x^2 = k^2$;

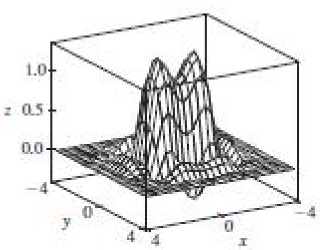


17. $x^2 + 9y^2 = k$; 



18. 

19. 

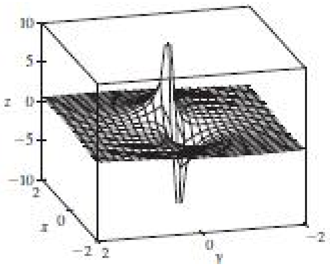
20. 

21. (a) C (b) Ⅱ 22. (a) A (b) Ⅳ 23. (a) F (b) Ⅰ 24. (a) E (b) Ⅲ

25. (a) B (b) Ⅵ 26. (a) D (b) Ⅴ

27. 평평한 평면족 28. 중심축이 x축인 원기둥족 $(k>0)$

29. (a) f의 그래프를 위쪽으로 2단위 이동한다.
    (b) f의 그래프를 수직방향으로 두 배로 늘린다.
    (c) f의 그래프를 xy평면에 대해 대칭이동한다.
    (d) f의 그래프를 xy평면에 대해 대칭이동하고 위쪽으로 2단위 이동한다.

30. 

; x, y가 커질수록 함숫값은 0으로 접근한다; (x, y)가 원점으로 접근할수록 f는 접근 방향에 따라 $\pm\infty$ 또는 0으로 접근한다.

31. $c=0$이면 그래프는 기둥면이다. $c>0$에 대해 등위곡선은 타원이다. 원점에서 멀어질수록 그래프는 위쪽으로 휘고, c가 증가할수록 가파름도 증가한다. $c<0$에 대해 등위곡선은 쌍곡선이다. 그래프는 y 방향에서 위로 향하고 x 방향에서는 아래로 향하며 xy평면에 접근한다. 그리고 $(0, 0, 1)$ 부근에서 안장점이 있다.

연습문제 11.2

01. 아무 것도 말할 수 없다.; f가 연속이면 $f(3, 1) = 6$이다. 02. 1

03. 존재하지 않는다. 04. 존재하지 않는다. 05. 0

06. 존재하지 않는다. 07. 2 08. 존재하지 않는다.

09. 그래프는 함수가 다른 직선을 따라 다른 수로 접근하는 것을 보인다.

10. $h(x, y) = (2x+3y-6)^2 + \sqrt{2x+3y-6}$; $\{(x, y) \mid 2x+3y \geq 6\}$

11. $\{(x, y) \mid x^2+y^2 \neq 1\}$ 12. $\{(x, y) \mid x^2+y^2 > 4\}$

13. $\{(x, y, z) \mid x^2+y^2+z^2 \leq 1\}$ 14. $\{(x, y) \mid (x, y) \neq (0, 0)\}$ 15. 0

16. -1 17. 생략

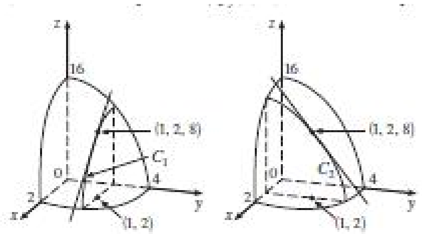
연습문제 11.3

01. (a) 위도와 시간이 고정되었을 때, 경도의 변화에 따른 온도의 변화율; 오직 위도의 변화에 대한 온도의 변화율; 오직 시간의 변화에 대한 온도의 변화율

(b) 양수, 음수, 양수

02. (a) 양수 (b) 음수

03. $f_x(1, 2) = -8 = C_1$의 기울기, $f_y(1, 2) = -4 = C_2$의 기울기;



04. $f_x(x, y) = -3y,\ f_y(x, y) = 5y^4 - 3x$

05. $f_x(x, t) = -\pi e^{-t}\sin\pi x,\ f_t(x, t) = -e^{-t}\cos\pi t$

06. $f_x(x, y) = 1/y,\ f_y(x, y) = -x/y^2$

07. $f_x(x, y) = \dfrac{(ad-bc)y}{(cx+dy)^2},\ f_y(x, y) = \dfrac{(bc-ad)x}{(cx+dy)^2}$

08. $g_u(u, v) = 10uv(u^2v-v^3)^4,\ g_v(u, v) = 5(u^2-3v^2)(u^2v-v^3)^4$

09. $R_p(p, q) = \dfrac{q^2}{1+p^2q^4},\ R_q(p, q) = \dfrac{2pq}{1+p^2q^4}$

10. $F_x(x, y) = \cos(e^x),\ F_y(x, y) = -\cos(e^y)$

11. $f_x = z - 10xy^3z^4,\ f_y = -15x^2y^2z^4,\ f_z = x - 20x^2y^3z^3$

12. $\dfrac{\partial w}{\partial x} = 1/(x+2y+3z),\ \dfrac{\partial w}{\partial y} = 2/(x+2y+3z),\ \dfrac{\partial w}{\partial z} = 3/(x+2y+3z)$

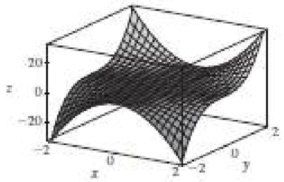
13. $\dfrac{\partial u}{\partial x} = y\sin^{-1}(yz),\ \dfrac{\partial u}{\partial y} = x\sin^{-1}(yz) + xyz/\sqrt{1-y^2z^2},\ \dfrac{\partial u}{\partial z} = xy^2/\sqrt{1-y^2z^2}$

14. $h_x = 2xy\cos(\frac{z}{t}),\ h_y = x^2\cos(\frac{z}{t}),\ h_z = (-x^2y/t)\sin(\frac{z}{t}),\ h_t = (x^2yz/t^2)\sin(\frac{z}{t})$

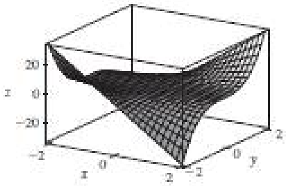
15. $\partial u/\partial x_i = x_i/\sqrt{x_1^2+x_2^2+\cdots+x_n^2}$

16. $\frac{1}{5}$ 17. $\frac{1}{4}$ 18. $f_x(x,\, y) = y^2 - 3x^2y,\ f_y(x,\, y) = 2xy - x^3$

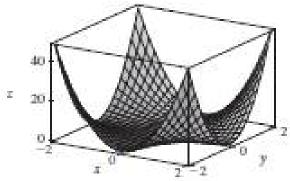
19.



$f(x,\, y) = x^2y^3$



$f_x(x,\, y) = 2xy^3$



$f_y(x,\, y) = 3x^2y^2$

20. $\frac{\partial z}{\partial x} = -\frac{x}{3z},\ \frac{\partial z}{\partial y} = -\frac{2y}{3z}$ 21. $\frac{\partial z}{\partial x} = \frac{yz}{e^z - xy},\ \frac{\partial z}{\partial y} = \frac{xz}{e^z - xy}$

22. (a) $f'(x),\, g'(y)$ (b) $f'(x+y),\, f'(x+y)$

23. $f_{xx} = 6xy^5 + 24x^2y,\ f_{xy} = 15x^2y^4 + 8x^3 = f_{yx},\ f_{yy} = 20x^3y^3$

24. $w_{uu} = v^2/(u^2+v^2)^{3/2},\ w_{uv} = -uv/(u^2+v^2)^{3/2} = w_{vu},\ w_{vv} = u^2/(u^2+v^2)^{3/2}$

25. $z_{xx} = -2x/(1+x^2)^2,\ z_{xy} = 0 = z_{yx},\ z_{yy} = -2y/(1+y^2)^2$

26. $u = x^4y^3 - y^4 \ \Rightarrow\ u_x = 4x^3y^3,\ u_{xy} = 12x^3y^2,\ u_y = 3x^4y^2 - 4y^3,\ u_{yx} = 12x^3y^2.$

따라서 $u_{xy} = u_{yx}$이다.

27. $24xy^2 - 6y,\ 24x^2y - 6x$ 28. $(2x^2y^2z^5 + 6xyz^3 + 2z)\,e^{xyz^2}$

29. $\theta e^{r\theta}\,(2\sin\theta + \theta\cos\theta + r\theta\sin\theta)$ 30. $6yz^2$

31. $u = e^{-\alpha^2k^2t}\sin kx \ \Rightarrow\ u_x = ke^{-\alpha^2k^2t}\cos kx,\ u_{xx} = -k^2e^{-\alpha^2k^2t}\sin kx,$

$u_t = -\alpha^2k^2e^{-\alpha^2k^2t}\sin kx$ 따라서 $\alpha^2u_{xx} = u_t$이다.

32. $u = \frac{1}{\sqrt{x^2+y^2+z^2}} \ \Rightarrow\ u_x = (-\tfrac{1}{2})(x^2+y^2+z^2)^{-3/2}(2x) = -x(x^2+y^2+z^2)^{-3/2}$

$u_{xx} = -(x^2+y^2+z^2)^{-3/2} - x(-\tfrac{3}{2})(x^2+y^2+z^2)^{-5/2}(2x) = \frac{2x^2-y^2-z^2}{(x^2+y^2+z^2)^{5/2}}$

$u_{yy} = \frac{2y^2-x^2-z^2}{(x^2+y^2+z^2)^{5/2}},\ u_{zz} = \frac{2z^2-x^2-y^2}{(x^2+y^2+z^2)^{5/2}}$

따라서 $u_{xx} + u_{yy} + u_{zz} = \frac{2x^2-y^2-z^2+2y^2-x^2-z^2+2z^2-x^2-y^2}{(x^2+y^2+z^2)^{5/2}} = 0$이다.

33. (a) $u = \sin(kx)\,\sin(akt) \Rightarrow\ u_t = ak\sin(kx)\,\cos(akt),\ u_{tt} = -a^2k^2\sin(kx)\,\sin(akt),$

$u_x = k\cos(kx)\,\sin(akt),\ u_{xx} = -k^2\sin(kx)\,\sin(akt)$

따라서 $u_{tt} = a^2u_{xx}$이다.

(b) $u = \dfrac{t}{a^2t^2 - x^2} \quad \Rightarrow \quad u_t = \dfrac{(a^2t^2 - x^2) - t(2a^2t)}{(a^2t^2 - x^2)^2} = -\dfrac{a^2t^2 + x^2}{(a^2t^2 - x^2)^2}.$

$$u_{tt} = \frac{-2a^2t(a^2t^2 - x^2)^2 + (a^2t^2 - x^2)(2)(a^2t^2 - x^2)(2a^2t)}{(a^2t^2 - x^2)^4} = \frac{2a^4t^3 + 6a^2tx^2}{(a^2t^2 - x^2)^3}.$$

$$u_x = t(-1)(a^2t^2 - x^2)^{-2}(2x) = \frac{2tx}{(a^2t^2 - x^2)^2}.$$

$$u_{xx} = \frac{2t(a^2t^2 - x^2)^2 - 2tx\,(2)(a^2t^2 - x^2)(-2x)}{(a^2t^2 - x^2)^4} = \frac{2a^2t^3 - 2tx^2 + 8tx^2}{(a^2t^2 - x^2)^3} = \frac{2a^2t^3 + 6tx^2}{(a^2t^2 - x^2)^3}.$$

따라서 $u_{tt} = a^2 u_{xx}$이다.

(c) $u = (x - at)^6 + (x + at)^6$

$$\Rightarrow \quad u_t = -6a(x - at)^5 + 6a(x + at)^5, \; u_{tt} = 30a^2(x - at)^4 + 30a^2(x + at)^4.$$

$$u_x = 6(x - at)^5 + 6(x + at)^5, \; u_{xx} = 30(x - at)^4 + 30(x + at)^4.$$

따라서 $u_{tt} = a^2 u_{xx}$이다.

(d) $u = \sin(x - at) + \ln(x + at)$

$$\Rightarrow \quad u_t = -a\cos(x - at) + \frac{a}{x + at}, \quad u_{tt} = -a^2\sin(x - at) - \frac{a^2}{(x + at)^2}.$$

$$u_x = \cos(x - at) + \frac{1}{x + at}, \quad u_{xx} = -\sin(x - at) - \frac{1}{(x + at)^2}$$

따라서 $u_{tt} = a^2 u_{xx}$이다.

34. $v = x + at, \; w = x - at$,

$$u_t = \frac{\partial[f(v) + g(w)]}{\partial t} = \frac{df(v)}{dv}\frac{\partial v}{\partial t} + \frac{dg(w)}{dw}\frac{\partial w}{\partial t} = af'(v) - ag'(w)$$

$$u_{tt} = \frac{\partial[af'(v) - ag'(w)]}{\partial t} = a[af''(v) + ag''(w)] = a^2[f''(v) + g''(w)]$$

$u_x = f'(v) + g'(w)$, $u_{xx} = f''(v) + g''(w)$, $u_{tt} = a^2 u_{xx}$

35. $z_x = e^y + ye^x, \; z_{xx} = ye^x, \; \partial^3 z/\partial x^3 = ye^x$

$z_y = xe^y + e^x, \; z_{yy} = xe^y, \; \partial^3 z/\partial y^3 = xe^y$

$\partial^3 z/\partial x \partial y^2 = e^y$, $\partial^3 z/\partial x^2 \partial y = e^x$

따라서 $z = x\,e^y + y\,e^x$는 주어진 편미분 방정식을 만족시킨다.

36. R^2/R_1^2

37. $\dfrac{\partial T}{\partial P}=\dfrac{V-nb}{nR}$, $\dfrac{\partial P}{\partial V}=\dfrac{2n^2a}{V^3}-\dfrac{nRT}{(V-nb)^2}$

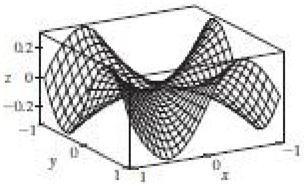
38. $\dfrac{\partial K}{\partial m}=\frac{1}{2}v^2$, $\dfrac{\partial K}{\partial v}=mv$, $\dfrac{\partial^2 K}{\partial v^2}=m$

따라서 $\dfrac{\partial K}{\partial m}\dfrac{\partial^2 K}{\partial v^2}=\dfrac{1}{2}v^2m=K$이다.

39. 아니다.　　　40. $x=1+t,\ y=2,\ z=2-2t$

41. 클레로의 정리에 의해 $f_{xyy}=(f_{xy})_y=(f_{yx})_y=f_{yxy}=(f_y)_{xy}=(f_y)_{yx}=f_{yyx}$이다.

42. -2

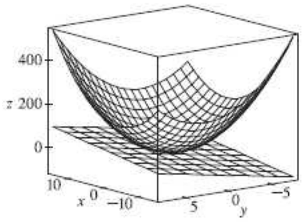
43. (a) 

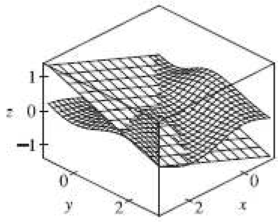
(b) $f_x(x,y)=\dfrac{x^4y+4x^2y^3-y^5}{(x^2+y^2)^2}$, $f_y(x,y)=\dfrac{x^5-4x^3y^2-xy^4}{(x^2+y^2)^2}$

(c) 0, 0　(d) 생략　(e) 아니다. f_{xy}와 f_{yx}가 연속이 아니기 때문이다.

연습문제 11.4

01. $z=-7x-6y+5$　　02. $x+y-2z=0$　　03. $x+y+z=0$

04. 

05. 

06. $6x+4y-23$　　07. $1-\pi y$

08. f의 선형 근사식은 $f(x,y)\approx f(0,0)+f_x(0,0)(x-0)+f_y(0,0)(y-0)=3+2x-12y$ 이다.

09. 6.3　　10. $\dfrac{3}{7}x+\dfrac{2}{7}y+\dfrac{6}{7}z$; 6.9914　　11. $dm=5p^4q^3\,dp+3p^5q^2\,dq$

12. $dR=\beta^2\cos\gamma\,d\alpha+2\alpha\beta\cos\gamma\,d\beta-\alpha\beta^2\sin\gamma\,d\gamma$

13. $\Delta z=0.9225,\ dz=0.9$　　14. $5.4\,\mathrm{cm}^2$　　15. $16\,\mathrm{cm}^3$

16. 2.3%　　17. $\dfrac{1}{17}\approx 0.059\,\Omega$　　18. $\varepsilon_1=\Delta x,\ \varepsilon_2=\Delta y$

19. 생략　　20. 생략

연습문제 11.5

01. $(2x+y)\cos t+(2y+x)e^t$

02. $e^{y/z}[2t-(x/z)-(2xy/z^2)]$

03. $\partial z/\partial s=2xy^3\cos t+3x^2y^2\sin t$, $\partial z/\partial t=-2sxy^3\sin t+3sx^2y^2\cos t$

04. $\dfrac{\partial z}{\partial s}=e^r\left(t\cos\theta-\dfrac{s}{\sqrt{s^2+t^2}}\sin\theta\right)$, $\dfrac{\partial z}{\partial t}=e^r\left(s\cos\theta-\dfrac{t}{\sqrt{s^2+t^2}}\sin\theta\right)$

05. 62   06. 7, 2

07. $\dfrac{\partial u}{\partial r}=\dfrac{\partial u}{\partial x}\dfrac{\partial x}{\partial r}+\dfrac{\partial u}{\partial y}\dfrac{\partial y}{\partial r}$, $\dfrac{\partial u}{\partial s}=\dfrac{\partial u}{\partial x}\dfrac{\partial x}{\partial s}+\dfrac{\partial u}{\partial y}\dfrac{\partial y}{\partial s}$, $\dfrac{\partial u}{\partial t}=\dfrac{\partial u}{\partial x}\dfrac{\partial x}{\partial t}+\dfrac{\partial u}{\partial y}\dfrac{\partial y}{\partial t}$

08. $\dfrac{\partial w}{\partial x}=\dfrac{\partial w}{\partial r}\dfrac{\partial r}{\partial x}+\dfrac{\partial w}{\partial s}\dfrac{\partial s}{\partial x}+\dfrac{\partial w}{\partial t}\dfrac{\partial t}{\partial x}$, $\dfrac{\partial w}{\partial y}=\dfrac{\partial w}{\partial r}\dfrac{\partial r}{\partial y}+\dfrac{\partial w}{\partial s}\dfrac{\partial s}{\partial y}+\dfrac{\partial w}{\partial t}\dfrac{\partial t}{\partial y}$

09. 1582, 3164, -700 10. 2π, -2π 11. $\dfrac{5}{144}$, $-\dfrac{5}{96}$, $\dfrac{5}{144}$

12. $\dfrac{1+x^4y^2+y^2+x^4y^4-2xy}{x^2-2xy-2x^5y^3}$ 13. $-\dfrac{x}{3z}$, $-\dfrac{2y}{3z}$

14. $\dfrac{yz}{e^z-xy}$, $\dfrac{xz}{e^z-xy}$ 15. 2℃/s   16. ≈ -0.33 (m/s)/min

17. (a) $6\,\mathrm{m^3/s}$ (b) $10\,\mathrm{m^2/s}$ (c) $0\,\mathrm{m/s}$ 18. ≈ -0.27 L/s

19. (a) $\partial z/\partial r=(\partial z/\partial x)\cos\theta+(\partial z/\partial y)\sin\theta$,
$\partial z/\partial\theta=-(\partial z/\partial x)r\sin\theta+(\partial z/\partial y)r\cos\theta$

(b) $\left(\dfrac{\partial z}{\partial r}\right)^2=\left(\dfrac{\partial z}{\partial x}\right)^2\cos^2\theta+2\dfrac{\partial z}{\partial x}\dfrac{\partial z}{\partial y}\cos\theta\sin\theta+\left(\dfrac{\partial z}{\partial y}\right)^2\sin^2\theta$,

$\left(\dfrac{\partial z}{\partial \theta}\right)^2=\left(\dfrac{\partial z}{\partial x}\right)^2 r^2\sin^2\theta-2\dfrac{\partial z}{\partial x}\dfrac{\partial z}{\partial y}r^2\cos\theta\sin\theta+\left(\dfrac{\partial z}{\partial y}\right)^2 r^2\cos^2\theta$.

따라서 $\left(\dfrac{\partial z}{\partial r}\right)^2+\dfrac{1}{r^2}\left(\dfrac{\partial z}{\partial \theta}\right)^2=\left[\left(\dfrac{\partial z}{\partial x}\right)^2+\left(\dfrac{\partial z}{\partial y}\right)^2\right](\cos^2\theta+\sin^2\theta)=\left(\dfrac{\partial z}{\partial x}\right)^2+\left(\dfrac{\partial z}{\partial y}\right)^2$ 이다.

20. $u=x-y$로 놓으면 $\dfrac{\partial z}{\partial x}=\dfrac{dz}{du}\dfrac{\partial u}{\partial x}=\dfrac{dz}{du}$ and $\dfrac{\partial z}{\partial y}=\dfrac{dz}{du}(-1)$이므로 $\dfrac{\partial z}{\partial x}+\dfrac{\partial z}{\partial y}=0$이다.

21. $\dfrac{\partial z}{\partial s}=\dfrac{\partial z}{\partial x}+\dfrac{\partial z}{\partial y}$, $\dfrac{\partial z}{\partial t}=\dfrac{\partial z}{\partial x}-\dfrac{\partial z}{\partial y}$ 이므로 $\dfrac{\partial z}{\partial s}\dfrac{\partial z}{\partial t}=\left(\dfrac{\partial z}{\partial x}\right)^2-\left(\dfrac{\partial z}{\partial y}\right)^2$ 이다.

22. $4rs\,\partial^2z/\partial x^2+(4r^2+4s^2)\,\partial^2z/\partial x\partial y+4rs\,\partial^2z/\partial y^2+2\,\partial z/\partial y$

23. $\dfrac{\partial z}{\partial r}=\dfrac{\partial z}{\partial x}\cos\theta+\dfrac{\partial z}{\partial y}\sin\theta$, $\dfrac{\partial z}{\partial \theta}=-\dfrac{\partial z}{\partial x}r\sin\theta+\dfrac{\partial z}{\partial y}r\cos\theta$

$$\frac{\partial^2 z}{\partial r^2}=\cos\theta\left(\frac{\partial^2 z}{\partial x^2}\cos\theta+\frac{\partial^2 z}{\partial y\,\partial x}\sin\theta\right)+\sin\theta\left(\frac{\partial^2 z}{\partial y^2}\sin\theta+\frac{\partial^2 z}{\partial x\,\partial y}\cos\theta\right)$$
$$=\cos^2\theta\,\frac{\partial^2 z}{\partial x^2}+2\cos\theta\,\sin\theta\,\frac{\partial^2 z}{\partial x\,\partial y}+\sin^2\theta\,\frac{\partial^2 z}{\partial y^2}$$

$$\frac{\partial^2 z}{\partial \theta^2} = -r\cos\theta\,\frac{\partial z}{\partial x} + (-r\sin\theta)\left(\frac{\partial^2 z}{\partial x^2}(-r\sin\theta) + \frac{\partial^2 z}{\partial y\,\partial x}\,r\cos\theta\right)$$

$$-r\sin\theta\,\frac{\partial z}{\partial y} + r\cos\theta\left(\frac{\partial^2 z}{\partial y^2}\,r\cos\theta + \frac{\partial^2 z}{\partial x\,\partial y}(-r\sin\theta)\right)$$

$$= -r\cos\theta\,\frac{\partial z}{\partial x} - r\sin\theta\,\frac{\partial z}{\partial y} + r^2\sin^2\theta\,\frac{\partial^2 z}{\partial x^2} - 2r^2\cos\theta\,\sin\theta\,\frac{\partial^2 z}{\partial x\,\partial y} + r^2\cos^2\theta\,\frac{\partial^2 z}{\partial y^2}$$

$$\frac{\partial^2 z}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2} + \frac{1}{r}\frac{\partial z}{\partial r} = (\cos^2\theta + \sin^2\theta)\,\frac{\partial^2 z}{\partial x^2} + (\sin^2\theta + \cos^2\theta)\,\frac{\partial^2 z}{\partial y^2}$$

$$-\frac{1}{r}\cos\theta\,\frac{\partial z}{\partial x} - \frac{1}{r}\sin\theta\,\frac{\partial z}{\partial y} + \frac{1}{r}\left(\cos\theta\,\frac{\partial z}{\partial x} + \sin\theta\,\frac{\partial z}{\partial y}\right)$$

$$= \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$$

24. 생략

연습문제 11.6

01. $2+\sqrt{3}/2$

02. (a) $\nabla f(x,\ y) = \langle 2\cos(2x+3y),\ 3\cos(2x+3y)\rangle$ (b) $\langle 2,\ 3\rangle$ (c) $\sqrt{3} - \dfrac{3}{2}$

03. (a) $\langle 2xyz - yz^3,\ x^2z - xz^3,\ x^2y - 3xyz^2\rangle$ (b) $\langle -3,\ 2,\ 2\rangle$ (c) $\dfrac{2}{5}$

04. $\dfrac{4-3\sqrt{3}}{10}$ 05. $-8\sqrt{10}$ 06. $4/\sqrt{30}$ 07. $2/5$

08. $1,\ \langle 0,\ 1\rangle$ 09. $1,\ \langle 3,\ 6,\ -2\rangle$ 10. (b) $\langle -12,\ 92\rangle$

11. 직선 $y = x+1$ 위의 모든 점 12. (a) $-40/(3\sqrt{3})$

13. (a) $32/\sqrt{3}$ (b) $\langle 38,\ 6,\ 12\rangle$ (c) $2\sqrt{406}$ 14. $\dfrac{327}{13}$

15. (a) $\nabla(au+bv) = \left\langle \dfrac{\partial(au+bv)}{\partial x}, \dfrac{\partial(au+bv)}{\partial y}\right\rangle = \left\langle a\dfrac{\partial u}{\partial x} + b\dfrac{\partial v}{\partial x}, a\dfrac{\partial u}{\partial y} + b\dfrac{\partial v}{\partial y}\right\rangle$

$$= a\left\langle \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right\rangle + b\left\langle \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}\right\rangle = a\nabla u + b\nabla v$$

(b) $\nabla(uv) = \left\langle v\dfrac{\partial u}{\partial x} + u\dfrac{\partial v}{\partial x}, v\dfrac{\partial u}{\partial y} + u\dfrac{\partial v}{\partial y}\right\rangle = v\left\langle \dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial y}\right\rangle + u\left\langle \dfrac{\partial v}{\partial x}, \dfrac{\partial v}{\partial y}\right\rangle$

$$= u\nabla v + v\nabla u$$

(c) $\nabla\left(\dfrac{u}{v}\right) = \left\langle \dfrac{v\dfrac{\partial u}{\partial x} - u\dfrac{\partial v}{\partial x}}{v^2}, \dfrac{v\dfrac{\partial u}{\partial y} - u\dfrac{\partial v}{\partial y}}{v^2}\right\rangle = \dfrac{v\left\langle \dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial y}\right\rangle - u\left\langle \dfrac{\partial v}{\partial x}, \dfrac{\partial v}{\partial y}\right\rangle}{v^2}$

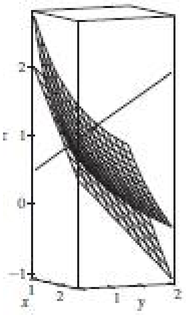
$$= \frac{v\nabla u - u\nabla v}{v^2}$$

(d) $\nabla u^n = \left\langle \frac{\partial(u^n)}{\partial x}, \frac{\partial(u^n)}{\partial y} \right\rangle = \left\langle nu^{n-1}\frac{\partial u}{\partial x}, nu^{n-1}\frac{\partial u}{\partial y} \right\rangle = nu^{n-1}\nabla u$

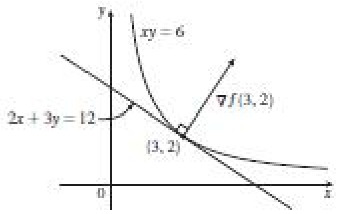
16. (a) $x+y+z=11$ (b) $x-3=y-3=z-5$

17. (a) $2x+3y+12z=24$ (b) $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{12}$

18. (a) $x+y+z=1$ (b) $x=y=z-1$

19. 

20. $\langle 2,\ 3\rangle$, $2x+3y=12$;



21. $\nabla F(x_0, y_0, z_0) = \left\langle \frac{2x_0}{a^2}, \frac{2y_0}{b^2}, \frac{2z_0}{c^2} \right\rangle$이고 $(x_0,\ y_0,\ z_0)$에서 접평면의 방정식은

$\frac{2x_0}{a^2}x + \frac{2y_0}{b^2}y + \frac{2z_0}{c^2}z = 2\left(\frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} + \frac{z_0^2}{c^2}\right) = 2(1) = 2$이므로 점 $(x_0,\ y_0,\ z_0)$은 타원면 위의 점이다. 이런 까닭에 접평면의 방정식은 $\frac{x\,x_0}{a^2}+\frac{y\,y_0}{b^2}+\frac{z\,z_0}{c^2}=1$이다.

22. 아니다. 23. $\left(-\frac{5}{4},\ -\frac{5}{4},\ \frac{25}{8}\right)$ 24. 생략

25. $x=-1-10t,\ y=1-16t,\ z=2-12t$ 26. 생략

27. $\mathrm{u}=\langle a,\ b\rangle$, $\mathrm{v}=\langle c,\ d\rangle$이면 af_x+bf_y와 cf_x+df_y가 알려진다. 따라서 f_x와 f_y에 대한 선형방정식을 푼다.

연습문제 11.7

01. (a) f는 $(1,1)$에서 극솟값을 갖는다. (b) f는 $(1,1)$에서 안장점을 갖는다.

02. 극솟값 $f\left(\frac{1}{3},\ -\frac{2}{3}\right)=-\frac{1}{3}$

03. 극댓값 $f(0,0)=2$, 극솟값 $f(0,4)=-30$, 안장점 $(2,2)$, $(-2,2)$

04. 극솟값 $f(2,1)=-8$, 안장점 $(0,0)$

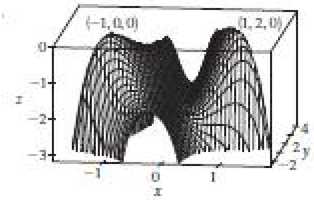
05. 없다. 06. 극솟값 $f(0,0)=0$, 안장점 $(\pm 1, 0)$

07. 극솟값 $f(0,1)=f(\pi,-1)=f(2\pi,1)=-1$, 안장점 $(\pi/2, 0)$, $(3\pi/2, 0)$

08. 극댓값 $f(0,0)=2$, 극솟값 $f(0,2)=-2$, 안장점 $(\pm 1, 1)$

09. 극댓값 $f(\pi/3, \pi/3)=3\sqrt{3}/2$, 극솟값 $f(5\pi/3, 5\pi/3)=-3\sqrt{3}/2$,

안장점 (π, π)

10. 극솟값 $f(0, -0.794) \approx -1.191$, $f(\pm 1.592, 1.267) \approx -1.310$,
    안장점 $(\pm 0.720, 0.259)$, 가장 낮은 점 $(\pm 1.592, 1.267, -1.310)$
11. 극댓값 $f(0.170, -1.215) \approx 3.197$,
    극솟값 $f(-1.301, 0.549) \approx -3.145$, $f(1.131, 0.549) \approx -0.701$,
    안장점 $(-1.301, -1.215)$, $(0.170, 0.549)$, $(1.131, -1.215)$
    가장 높거나 가장 낮은 점은 없다.
12. 극댓값 $f(0, \pm 2) = 4$, 극솟값 $f(1, 0) = -1$
13. 극댓값 $f(\pm 1, 1) = 7$, 극솟값 $f(0, 0) = 4$
14. 극댓값 $f(3, 0) = 83$, 극솟값 $f(1, 1) = 0$
15. 

16. $2/\sqrt{3}$　　17. $(2, 1, \sqrt{5})$, $(2, 1, -\sqrt{5})$

18. $\frac{100}{3}$, $\frac{100}{3}$, $\frac{100}{3}$　　19. $8r^3/(3\sqrt{3})$　　20. $\frac{4}{3}$

21. 정육면체, 변의 길이 $c/12$
22. 밑면은 변의 길이가 $40\,\text{cm}$인 정사각형이고 높이가 $20\,\text{cm}$
23. $L^3/(3\sqrt{3})$　　24. 생략

연습문제 11.8

01. 최댓값 없음, 최솟값 $f(1, 1) = f(-1, -1) = 2$
02. 최댓값 $f(\pm 2, 1) = 4$, 최솟값 $f(\pm 2, -1) = -4$
03. 최댓값 $f(1, 3, 5) = 70$, 최솟값 $f(-1, -3, -5) = -70$
04. 최댓값 $2/\sqrt{3}$, 최솟값은 $-2/\sqrt{3}$
05. 최댓값 $\sqrt{3}$, 최솟값 1
06. 최댓값 $f\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) = 2$, 최솟값 $f\left(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right) = -2$
07. 최댓값 $f(1, \sqrt{2}, -\sqrt{2}) = 1 + 2\sqrt{2}$, 최솟값 $f(1, -\sqrt{2}, \sqrt{2}) = 1 - 2\sqrt{2}$
08. 최댓값 $\frac{3}{2}$, 최솟값 $\frac{1}{2}$
09. 최댓값 $f(3/\sqrt{2}, -3/\sqrt{2}) = 9 + 12\sqrt{2}$, 최솟값 $f(-2, 2) = -8$
10. 최댓값 $f(\pm 1/\sqrt{2}, \mp 1/(2\sqrt{2})) = e^{1/4}$, 최솟값 $f(\pm 1/\sqrt{2}, \pm 1/(2\sqrt{2})) = e^{-1/4}$
11. $\approx 59, 30$　　12. (a) 생략 (b) 생략 (c) 생략　　13. 생략　　14. 생략
15~20. 11.7절 [연습문제 16~21] 참조.　　21. $L^3/(3\sqrt{3})$

22. 가장 가까운 점: $\left(\frac{1}{2},\ \frac{1}{2},\ \frac{1}{2}\right)$, 가장 먼 점: $(-1,\ -1,\ 2)$

23. 최댓값: ≈ 9.7938, 최솟값: ≈ -5.3506

24. (a) c/n (b) $x_1 = x_2 = \cdots = x_n$일 때

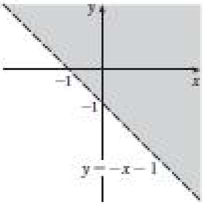
11장 복습문제

참-거짓 질문

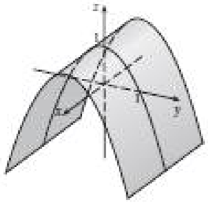
01. 참 02. 거짓 03. 거짓 04. 참 05. 거짓 06. 참

연습문제

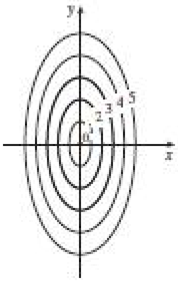
01. $\{(x,\ y) \mid y > -x-1\}$;



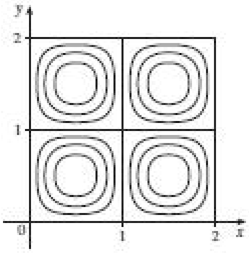
02.



03.



04.



05. $\frac{2}{3}$

06. $f_x = 32xy(5y^3 + 2x^2y)^7,\ f_y = (16x^2 + 120y^2)(5y^3 + 2x^2y)^7$

07. $F_\alpha = \dfrac{2\alpha^3}{\alpha^2 + \beta^2} + 2\alpha \ln(\alpha^2 + \beta^2),\ F_\beta = \dfrac{2\alpha^2\beta}{\alpha^2 + \beta^2}$

08. $S_u = \arctan(v\sqrt{w}),\ S_v = \dfrac{u\sqrt{w}}{1 + v^2w},\ S_w = \dfrac{uv}{2\sqrt{w}(1 + v^2w)}$

09. $f_{xx} = 24x,\ f_{xy} = -2y = f_{yx},\ f_{yy} = -2x$

10. $f_{xx} = k(k-1)x^{k-2}y^lz^m,\ f_{xy} = klx^{k-1}y^{l-1}z^m = f_{yx},$

$f_{xz} = kmx^{k-1}y^lz^{m-1} = f_{zx},\ f_{yy} = l(l-1)x^ky^{l-2}z^m,$

$f_{yz} = lmx^ky^{l-1}z^{m-1} = f_{zy},\ f_{zz} = m(m-1)x^ky^lz^{m-2}$

11. $z=x\,y+x\,e^{y/x} \;\Rightarrow\; \dfrac{\partial z}{\partial x}=y-\dfrac{y}{x}e^{y/x}+e^{y/x},\ \dfrac{\partial z}{\partial y}=x+e^{y/x}$

$$x\,\frac{\partial z}{\partial x}+y\,\frac{\partial z}{\partial y}=x\left(y-\frac{y}{x}e^{y/x}+e^{y/x}\right)+y\left(x+e^{y/x}\right)=xy-ye^{y/x}+xe^{y/x}+xy+ye^{y/x}$$

$$=xy+xy+xe^{y/x}=x\,y+z$$

12. (a) $z=8x+4y+1$ (b) $\dfrac{x-1}{8}=\dfrac{y+2}{4}=\dfrac{z-1}{-1}$

13. (a) $2x-2y-3z=3$ (b) $\dfrac{x-2}{4}=\dfrac{y+1}{-4}=\dfrac{z-1}{-6}$

14. (a) $x+2y+5z=0$ (b) $x-2=\dfrac{y+1}{2}=\dfrac{z}{5}$

15. $\left(2,\ \dfrac{1}{2},\ -1\right),\ \left(-2,\ -\dfrac{1}{2},\ 1\right)$

16. $60x+\dfrac{24}{5}y+\dfrac{32}{5}z-120$; 38.656

17. $2xy^3(1+6p)+3x^2y^2(p\,e^p+e^p)+4z^3(p\cos p+\sin p)$

18. $-47,\ 108$

19. $f'=\dfrac{df}{d(x^2-y^2)},\ \dfrac{\partial z}{\partial x}=2xf'(x^2-y^2),\ \dfrac{\partial z}{\partial y}=1-2yf'(x^2-y^2)$

$$y\,\frac{\partial z}{\partial x}+x\,\frac{\partial z}{\partial y}=2xyf'(x^2-y^2)+x-2xyf'(x^2-y^2)=x.$$

20. $\dfrac{\partial z}{\partial x}=\dfrac{\partial z}{\partial u}\,y+\dfrac{\partial z}{\partial v}\dfrac{-y}{x^2}$

$$\frac{\partial^2 z}{\partial x^2}=y\,\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial u}\right)+\frac{2y}{x^3}\frac{\partial z}{\partial v}+\frac{-y}{x^2}\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial v}\right)$$

$$=\frac{2y}{x^3}\frac{\partial z}{\partial v}+y\left(\frac{\partial^2 z}{\partial u^2}\,y+\frac{\partial^2 z}{\partial v\,\partial u}\frac{-y}{x^2}\right)+\frac{-y}{x^2}\left(\frac{\partial^2 z}{\partial v^2}\frac{-y}{x^2}+\frac{\partial^2 z}{\partial u\,\partial v}\,y\right)$$

$$=\frac{2y}{x^3}\frac{\partial z}{\partial v}+y^2\,\frac{\partial^2 z}{\partial u^2}-\frac{2y^2}{x^2}\frac{\partial^2 z}{\partial u\,\partial v}+\frac{y^2}{x^4}\frac{\partial^2 z}{\partial v^2}$$

$$\frac{\partial z}{\partial y}=x\,\frac{\partial z}{\partial u}+\frac{1}{x}\frac{\partial z}{\partial v}$$

$$\frac{\partial^2 z}{\partial y^2}=x\,\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial u}\right)+\frac{1}{x}\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial v}\right)=x\left(\frac{\partial^2 z}{\partial u^2}x+\frac{\partial^2 z}{\partial v\,\partial u}\frac{1}{x}\right)+\frac{1}{x}\left(\frac{\partial^2 z}{\partial v^2}\frac{1}{x}+\frac{\partial^2 z}{\partial u\,\partial v}\,x\right)$$

$$=x^2\,\frac{\partial^2 z}{\partial u^2}+2\,\frac{\partial^2 z}{\partial u\,\partial v}+\frac{1}{x^2}\frac{\partial^2 z}{\partial v^2}$$

$$x^2\,\frac{\partial^2 z}{\partial x^2}-y^2\,\frac{\partial^2 z}{\partial y^2}$$

$$= \frac{2y}{x}\frac{\partial z}{\partial v} + x^2y^2\frac{\partial^2 z}{\partial u^2} - 2y^2\frac{\partial^2 z}{\partial u\,\partial v} + \frac{y^2}{x^2}\frac{\partial^2 z}{\partial v^2} - x^2y^2\frac{\partial^2 z}{\partial u^2} - 2y^2\frac{\partial^2 z}{\partial u\,\partial v} - \frac{y^2}{x^2}\frac{\partial^2 z}{\partial v^2}$$

$$= \frac{2y}{x}\frac{\partial z}{\partial v} - 4y^2\frac{\partial^2 z}{\partial u\,\partial v} = 2v\frac{\partial z}{\partial v} - 4uv\frac{\partial^2 z}{\partial u\,\partial v}$$ ($y = xv = \frac{uv}{y}$ 또는 $y^2 = uv$이므로)

21. $\langle 2x\,e^{yz^2},\ x^2z^2e^{yz^2},\ 2x^2yz\,e^{yz^2} \rangle$ 22. $-\frac{4}{5}$ 23. $\sqrt{145}/2,\ \left\langle 4,\ \frac{9}{2} \right\rangle$

24. 최솟값: $f(-4,1) = -11$

25. 최댓값: $f(1,1) = 1$, 안장점: $(0,0)$, $(0,3)$, $(3,0)$

26. 최댓값: $f(1,2) = 4$, 최솟값: $f(2,4) = -64$

27. 최댓값: $f(-1,0) = 2$, 최솟값: $f(1,\pm 1) = -3$, 안장점: $(-1,\pm 1)$, $(1,0)$

28. 최댓값: $f(\pm\sqrt{2/3},\ 1/\sqrt{3}) = 2/(3\sqrt{3})$,

최솟값: $f(\pm\sqrt{2/3},\ -1/\sqrt{3}) = -2/(3\sqrt{3})$

29. 최댓값: 1, 최솟값: -1

30. $(\pm 3^{-1/4},\ 3^{-1/4}\sqrt{2},\ \pm 3^{1/4})$, $(\pm 3^{-1/4},\ -3^{-1/4}\sqrt{2},\ \pm 3^{1/4})$

31. $P(2-\sqrt{3})$, $P(3-\sqrt{3})/6$, $P(2\sqrt{3}-3)/3$
