

연습문제 해답

게시 일자 : 2018- 03- 30

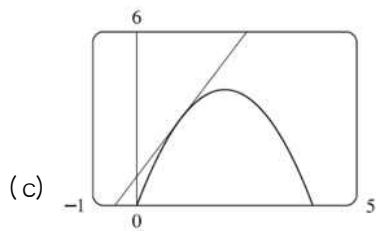
3장

도함수

3.1 미분계수와 변화율

01. (a) 2

(b) $y = 2x + 1$



02. $y = -8x + 12$

03. (a) $8a - 6a^2$

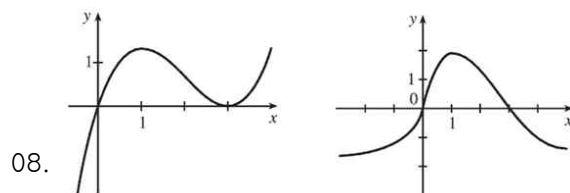
(b) $y = 2x + 3, y = -8x + 19$

04. -9.6 m/s

05. $v(a) = \frac{-2}{a^3} \text{ m/s}, v(1) = -2 \text{ m/s}, v(2) = -\frac{1}{4} \text{ m/s}, v(3) = -\frac{2}{27} \text{ m/s}$

06. $g'(0) < 0 < g'(4) < g'(2) < g'(-2)$

07. $f(2) = 3, f'(2) = 4$



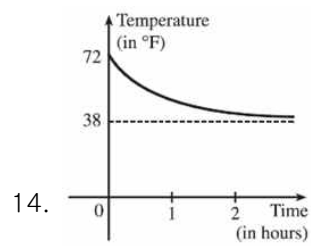
09. $y = 3x - 1$

10. $\frac{5}{(a+3)^2}$

11. $\frac{-1}{\sqrt{1-2a}}$

12. $f(x) = 2^x$ and $a = 5$

13. $f(x) = \cos x$ and $a = \pi$ 또는 $f(x) = \cos(\pi + x)$ and $a = 0$



15.

(a) 생산한 금괴의 kg당 달러의 변화율

(b) 50kg의 금이 생산되었을 때, 생산비용이 36달러/kg이다.

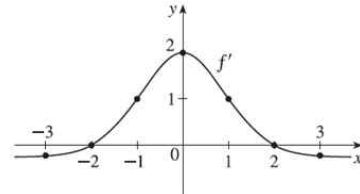
(c) 단기간에 감소하고 장기간에 증가한다.

16. 존재하지 않는다.

3.2 함수로서의 도함수

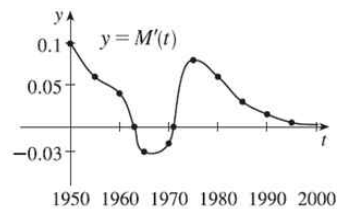
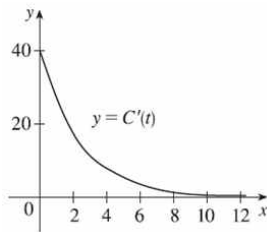
01.

- (a) $f'(-3) \approx -0.2$ (b) $f'(-2) \approx 0$ (c) $f'(-1) \approx 1$ (d) $f'(0) \approx 2$
 (e) $f'(1) \approx 1$ (f) $f'(2) \approx 0$ (g) $f'(3) \approx -0.2$



02.

- (a) 경과시간(h)에 관한 총용량의 백분율에 대한 순간변화율
 (b) 총용량의 백분율에 대한 변화율이 감소하여 0으로 접근한다.

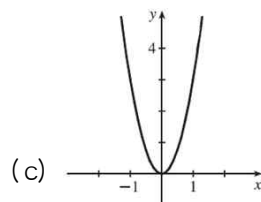


03. 1963년에서 1971년까지

04. (a) 0, 1, 2, 4 (b) -1, -2, -4 (c) $f'(x) = 2x$ (d) $f'(x) = 2x$

05. (a) 0, 0.75, 3, 12, 27

- (b) 0.75, 3, 12, 27



- (d) $f'(x) = 3x^2$ (e) $f'(x) = 3x^2$

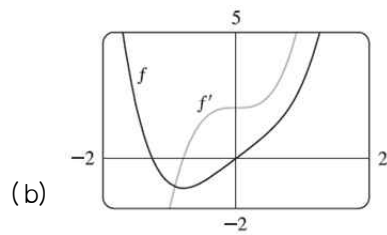
06. $f'(x) = \frac{1}{2}$, \square , \square

07. $f'(x) = 3x^2 - 3$, \square , \square

08. $g'(x) = \frac{-1}{\sqrt[2]{9-x}}$, $(-\infty, 9]$, $(-\infty, 9)$

09. $G'(t) = \frac{4}{(t+1)^2}$, $(-\infty, -1) \cup (-1, \infty)$

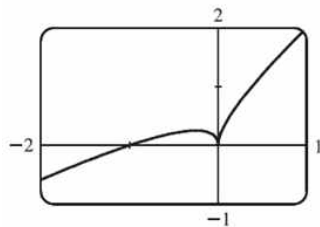
10. (a) $f'(x) = 4x^3 + 2$



11. -4 (뾰족한 점), 0 (불연속 점)

12. -1 (수직접선); 4 (뾰족한 점)

13. -1 에서 미분 가능, 0 에서 미분 불가능



14. $a = f$, $b = f'$, $c = f''$

15. $a =$ 가속도, $b =$ 속도, $c =$ 위치

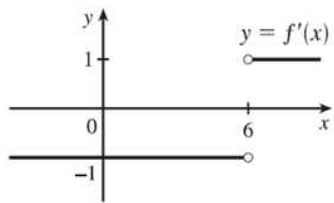
16.

(a) $\frac{1}{3a^{2/3}}$ or $\frac{1}{3}a^{-2/3}$

(b) $f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt[3]{h} - 0}{h} = \lim_{h \rightarrow 0} \frac{1}{h^{2/3}}$

(c) $\lim_{x \rightarrow 0} |f'(x)| = \lim_{x \rightarrow 0} \frac{1}{3x^{2/3}} = \infty$ and f is continuous at $x = 0$

17. $f'(x) = \begin{cases} 1 & \text{if } x > 6 \\ -1 & \text{if } x < 6 \end{cases} \Leftrightarrow f'(x) = \frac{x-6}{|x-6|}$



3.3 기본적인 미분 공식

01. $f'(x) = 0$

02. $f(x) = x^3 - 4x + 6 \Rightarrow f'(x) = 3x^2 - 4(1) + 0 = 3x^2 - 4$

03. $g'(x) = 2x - 6x^2$

04. $A(s) = -\frac{12}{s^5} = -12s^{-5} \Rightarrow A'(s) = -12(-5s^{-6}) = 60s^{-6} \text{ or } 60/s^6$

05. $S'(p) = \frac{1}{2}p^{-1/2} - 1$ 또는 $S'(p) = \frac{1}{\sqrt{2p}} - 1$

06. $y' = \frac{3}{2}x^{1/2} + 4(\frac{1}{2})x^{-1/2} + 3(-\frac{1}{2})x^{-3/2} = \frac{3}{2}\sqrt{x} + \frac{2}{\sqrt{x}} - \frac{3}{2x\sqrt{x}}$

07. $\frac{dz}{dy} = A(-10)y^{-11} + B(-\sin y) = -\frac{10A}{y^{11}} - B\sin y$

08. $y = -3\sqrt{3}x + 3 + \pi\sqrt{3}, y = \frac{x}{3\sqrt{3}} + 3 - \frac{\pi}{9\sqrt{3}}$

09. $g(t) = 2\cos t - 3\sin t \Rightarrow g'(t) = -2\sin t - 3\cos t \Rightarrow g''(t) = -2\cos t + 3\sin t$

10. $-\cos x$

11. $(2n+1)\pi \pm \frac{\pi}{3}, n \text{ 은 정수}$

12.

(a) $s = t^3 - 3t \Rightarrow v(t) = s'(t) = 3t^2 - 3 \Rightarrow a(t) = v'(t) = 6t$

(b) $a(2) = 6(2) = 12 \text{ m/s}^2$

(c) $v(t) = 3t^2 - 3 = 0$ when $t^2 = 1$, that is, $t = 1$ [$t \geq 0$] and $a(1) = 6 \text{ m/s}^2$.

13. (a) $t = 4\text{s}$ (b) $t = 1.5\text{s}$, 속도가 최소

14.

(a) $S'(20) = 160\pi \text{ cm}^2/\text{cm}$

(b) $S'(40) = 320\pi \text{ cm}^2/\text{cm}$

(c) $S'(60) = 480\pi \text{ cm}^2/\text{cm}$

15.

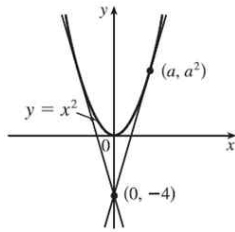
(a) $V = \frac{5.3}{P}$

(b) -0.00212 , 25°C 에서 압력에 관한 부피의 순간변화율, m^3/kPa

16.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \rightarrow 0} \frac{x - (x+h)}{hx(x+h)} = \lim_{h \rightarrow 0} \frac{-h}{hx(x+h)} = \lim_{h \rightarrow 0} \frac{-1}{x(x+h)} = -\frac{1}{x^2}$$

17. $(2, 4)$ 와 $(-2, 4)$



18. $a = -\frac{1}{2}$, $b = 2$

19. $y = 2x^2 - x$

20. 1000

21.

[3]

$$\frac{d}{dx}(\sin x) = \cos x$$

PROOF If $f(x) = \sin x$, then

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h} \\ &= \lim_{h \rightarrow 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) \right] \\ &= \lim_{h \rightarrow 0} \sin x \cdot \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} + \lim_{h \rightarrow 0} \cos x \cdot \lim_{h \rightarrow 0} \frac{\sin h}{h} \\ &= (\sin x) \cdot 0 + (\cos x) \cdot 1 = \cos x \end{aligned}$$

□

[4]

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$f(x) = \cos x \Rightarrow$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h} = \lim_{h \rightarrow 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h} \\ &= \lim_{h \rightarrow 0} \left(\cos x \frac{\cos h - 1}{h} - \sin x \frac{\sin h}{h} \right) = \cos x \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} - \sin x \lim_{h \rightarrow 0} \frac{\sin h}{h} \\ &= (\cos x)(0) - (\sin x)(1) = -\sin x \end{aligned}$$

3.4 곱과 나눗셈의 법칙

01. $1 - 2x + 6x^2 - 8x^3$

02. $g'(t) = t^3(-\sin t) + (\cos t) \cdot 3t^2 = 3t^2 \cos t - t^3 \sin t$ or $t^2(3 \cos t - t \sin t)$

03. $f'(x) = \cos x - \frac{1}{2} \csc^2 x$

04. $g'(x) = \frac{(3-4x)(2) - (1+2x)(-4)}{(3-4x)^2} = \frac{6-8x+4+8x}{(3-4x)^2} = \frac{10}{(3-4x)^2}$

05. $y' = 2v - 2\left(\frac{1}{2}\right)v^{-1/2} = 2v - v^{-1/2}$

06. $y' = \frac{(2-\tan x)(1) - x(-\sec^2 x)}{(2-\tan x)^2} = \frac{2-\tan x + x \sec^2 x}{(2-\tan x)^2}$

07.

$$y' = \frac{(1+t)(t \cos t + \sin t) - t \sin t(1)}{(1+t)^2} = \frac{t \cos t + \sin t + t^2 \cos t + t \sin t - t \sin t}{(1+t)^2} = \frac{(t^2 + t) \cos t + \sin t}{(1+t)^2}$$

08. $y = \frac{2}{3}x - \frac{2}{3}$

09. $y = x - \pi - 1$

10.

$$\frac{d}{dx}(\csc x) = \frac{d}{dx}\left(\frac{1}{\sin x}\right) = \frac{(\sin x)(0) - 1(\cos x)}{\sin^2 x} = \frac{-\cos x}{\sin^2 x} = -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} = -\csc x \cot x$$

11.

$$\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{(\cos x)(0) - 1(-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \sec x \tan x$$

12.

$$\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) = \frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{\sin^2 x} = -\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\csc^2 x$$

13. (a) -16

(b) $-\frac{20}{9}$

(c) 20

14. $2\mathbb{H}, (-2 \pm \sqrt{3}, \frac{1 \mp \sqrt{3}}{2})$

15. -0.2436 K/min

16.

(a) $(fgh)' = [(fg)h]' = (fg)'h + (fg)h' = (f'g + fg')h + (fg)h' = f'gh + fg'h + fgh'$

(b) $y = x \sin x \cos x \Rightarrow \frac{dy}{dx} = \sin x \cos x + x \cos x \cos x + x \sin x (-\sin x) = \sin x \cos x + x \cos^2 x - x \sin^2 x$

17.

(a) 생략

(b) $y = -\frac{2x(2x^2+1)}{(x^4+x^2+1)^2}$

(c) 생략

3.5 연쇄법칙

01. $\frac{4}{3\sqrt[3]{(1+4x)^2}}$

02. $\frac{\cos x}{2\sqrt{\sin x}}$

03. $10x(x^4 + 3x^2 - 2)^4(2x^2 + 3)$

04. $-\frac{2z}{(z^2 + 1)^2}$

05. 곱의 법칙 사용, $\sec kx (kx \tan kx + 1)$

06. $\frac{2}{3}(t+1)^{-1/3}(2t^2-1)^2(20t^2+18t-1)$

07. $(\cos x - x \sin x) \cos(x \cos x)$

08. $(x \cos \sqrt{1+x^2})/\sqrt{1+x^2}$

09. $4 \sec^2 x \tan x$

10. $-2 \cos \theta \cot(\sin \theta) \csc^2(\sin \theta)$

11. $p(2r \sin rx + n)^{p-1}(2r^2 \cos rx)$

12. 1계 도함수 $-2x \sin(x^2)$, 2계 도함수 $-4x^2 \cos(x^2) - 2 \sin(x^2)$

13. $y = -x + \pi$

14. $(\frac{\pi}{2} + 2n\pi, 3)$ and $(\frac{3\pi}{2} + 2n\pi, -1)$, n 은 정수

15.

(a) $h(x) = f(g(x)) \Rightarrow h'(x) = f'(g(x)) \cdot g'(x)$, so $h'(1) = f'(g(1)) \cdot g'(1) = f'(2) \cdot 6 = 5 \cdot 6 = 30$.

(b) $H(x) = g(f(x)) \Rightarrow H'(x) = g'(f(x)) \cdot f'(x)$, so $H'(1) = g'(f(1)) \cdot f'(1) = g'(3) \cdot 4 = 9 \cdot 4 = 36$.

16. $-\frac{1}{3\sqrt{2}}$ or $-\frac{1}{6}\sqrt{2}$

17. $-2^{50} \cos 2x$

18.

(a) $B(t) = 4.0 + 0.35 \sin \frac{2\pi t}{5.4} \Rightarrow \frac{dB}{dt} = \left(0.35 \cos \frac{2\pi t}{5.4}\right) \left(\frac{2\pi}{5.4}\right) = \frac{0.7\pi}{5.4} \cos \frac{2\pi t}{5.4} = \frac{7\pi}{54} \cos \frac{2\pi t}{5.4}$

(b) At $t = 1$, $\frac{dB}{dt} = \frac{7\pi}{54} \cos \frac{2\pi}{5.4} \approx 0.16$.

19. 생략

20. (a) 생략

(b) $V = \frac{4}{3}\pi r^3, \frac{dV}{dt} = \frac{dV}{dr} \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$

21. Since $\theta^\circ = \left(\frac{\pi}{180}\right)\theta$ rad, we have $\frac{d}{d\theta} (\sin \theta^\circ) = \frac{d}{d\theta} \left(\sin \frac{\pi}{180}\theta\right) = \frac{\pi}{180} \cos \frac{\pi}{180}\theta = \frac{\pi}{180} \cos \theta^\circ$.

3.6 음함수의 미분법

01. (a) $\frac{9x}{y}$ (b) $\pm \frac{9x}{\sqrt{9x^2 - 1}}$ (c) 생략

02. $-\frac{x(3x+2y)}{x^2+8y}$

03. $\frac{2x+y\sin x}{\cos x-2y}$

04. $\frac{4xy\sqrt{xy}-y}{x-2x^2\sqrt{xy}}$

05. $\frac{y\sin x+y\cos(xy)}{\cos x-x\cos(xy)}$

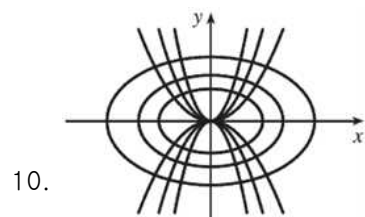
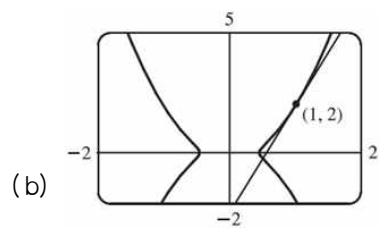
06. $y = x + \frac{1}{2}$

07. $y = -\frac{9}{13}x + \frac{40}{13}$

08. $-\frac{2x}{y^5}$

09.

(a) $y = \frac{9}{2}x - \frac{5}{2}$



11. (a) $\frac{V^3(nb - V)}{PV^3 - n^2aV + 2n^3ab}$ (b) -4.04 L/atm

12. $(\pm\sqrt{3}, 0)$

13. 생략

14. (a) 0 (b) $-\frac{1}{2}$

3.7 역함수의 도함수

01. $\frac{\ln x + 2}{2\sqrt{x}}$

02. $\frac{\cos(\ln x)}{x}$

03. $\frac{-2a}{a^2 - x^2}$

04. $-\frac{1}{x}$

05. $e^x(x^3 + 3x^2 + 2x + 2)$

06. $3ax^2e^{ax^3}$

07. $e^{t \sin 2t}(2t \cos 2t + \sin 2t)$

08. e^{e^x+x}

09. $5x^4 + 5^x \ln 5$

10. $x^{\sin x} \left(\frac{\sin x}{x} + \ln x \cos x \right)$

11. $(\cos x)^x (\ln \cos x - x \tan x)$

12. $\frac{2 \tan^{-1} x}{1 + x^2}$

13. $-\frac{\sin \theta}{1 + \cos^2 \theta}$

14. $\frac{1}{\sqrt{-x^2 - x}}$

15. $\tanh x$

$$16. 3e^{\cosh 3x} \sinh 3x$$

$$17. \frac{1}{2\sqrt{x(x-1)}}$$

$$18. -\csc x$$

$$19. x + 2x \ln(2x)$$

$$20. \frac{2x}{x^2 + y^2 - 2y}$$

$$21. \frac{y(y - e^{x/y})}{y^2 - xe^{x/y}}$$

$$22. y = 2x - 2$$

$$23. y = 2x + 1$$

$$24. y = (10 \ln 10)x + 10(1 - \ln 10)$$

$$25. (x^2 + 2)^2(x^4 + 4)^4 \left(\frac{4x}{x^2 + 2} + \frac{16x^3}{x^4 + 4} \right)$$

$$26. \sqrt{\frac{x-1}{x^4+1}} \left(\frac{1}{2x-2} - \frac{2x^3}{x^4+1} \right)$$

$$27. f^{(n)}(x) = 2^n e^{2x}$$

$$28. f^{(n)}(x) = (-1)^{n-1} \cdot 2 \cdot 3 \cdot 4 \cdot \dots \cdot (n-1)(x-1)^{-n} = (-1)^{n-1} \frac{(n-1)!}{(x-1)^n}$$

$$29. -2 \text{ 또는 } -4$$

$$30. \text{(a) } 0.3572$$

$$\text{(b) } 70.34^\circ$$

3.8 관련된 비율

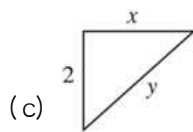
01. $3x^2 \frac{dx}{dt}$

02. $\frac{3}{25\pi} \text{ m/min}$

03. -18

04. (a) 800 km/h

(b) 3 km



(d) $2y \left(\frac{dy}{dt} \right) = 2x \left(\frac{dx}{dt} \right)$

(e) 596 km/h

05. 2.79 m/s

06. 55.4 km/h

07. $\frac{1}{3} \text{ m/min}$

08. $0.3 \text{ m}^2/\text{s}$

09. $80 \text{ cm}^3/\text{min}$

10. $0.132 \Omega/\text{s}$

11. 296 km/h

12. (a) $P(t) = 100e^{(\ln 4.2)t} = 100(4.2)^t$

(b) 약 7409

(c) $(\ln 4.2)(100(4.2)^3)$

(d) 3.2 시간

13. (a) $100 \cdot 2^{-t/30} \text{ mg}$

(b) 9.92 mg

(c) 199.3년

14. (a) 58°C

(b) 98분

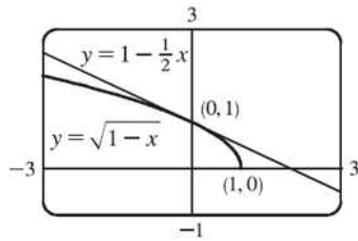
15. (a) 64.5 kPa

(b) 39.9 kPa

3.9 선형 근사와 미분

01. $-10x - 6$

02. $\sqrt{1-x} = 1 - \frac{1}{2}x$, $\sqrt{0.9} = 0.95$, $\sqrt{0.99} = 0.995$



03. $-0.368 < x < 0.677$

04. 15.968

05.

(a) $\sec^2 x dx$

(b) $dy = -0.2$

(c) $\Delta y \approx -0.18237$

06.

(a) 최대 허용 오차 = 270 cm^3 , 상대 오차 = 0.01, 백분율 오차 = 1%

(b) 최대 허용 오차 = 36 cm^2 , 상대 오차 = $0.00\bar{6}$, 백분율 오차 = $0.\bar{6}\%$

07. 생략

08.

(a) 4.8 and 5.2

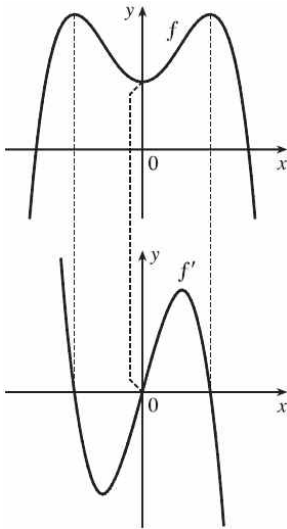
(b) 생략

3장 복습문제

연습문제

01. $f''(5) < 0 < f'(5) < f'(2) < 1 < f'(3)$

02.



03. $3x^2 + 5$

04. $4x^7(x+1)^3(3x+2)$

05. $x(\pi x \cos \pi x + 2 \sin \pi x)$

06. $-\frac{\sec^2 \sqrt{1-x}}{2\sqrt{1-x}}$

07. $\frac{2 \sec 2\theta (\tan 2\theta - 1)}{(1 + \tan 2\theta)^2}$

08. $\frac{2x - y \cos(xy)}{x \cos(xy) + 1}$

09. $\frac{\cos \sqrt{x} - \sqrt{x} \sin \sqrt{x}}{2\sqrt{x}}$

10. $\frac{1}{5}(x \tan x)^{-4/5}(\tan x + x \sec^2 x)$

$$11. \frac{1 + \ln x}{x \ln x}$$

$$12. \frac{1}{2 \sqrt{\arctan x} (1 + x^2)}$$

$$13. \frac{-e^{1/x}(1 + 2x)}{x^4}$$

$$14. 3^{x \ln x} (\ln 3)(1 + \ln x)$$

$$15. 2 \sec^2(2\theta) e^{\tan 2\theta}$$

$$16. \frac{2}{(1 + 2x) \ln 5}$$

$$17. \frac{4x}{1 + 16x^2} + \tan^{-1}(4x)$$

$$18. 3 \tanh 3x$$

$$19. (\cosh x) / \sqrt{\sinh^2 x - 1}$$

$$20. \frac{(2 - x)^4(3x^2 - 55x - 52)}{2 \sqrt{x + 1} (x + 3)^8}$$

$$21. -\frac{5x^4}{y^{11}}$$

$$22. y = 2\sqrt{3}x + 1 - \pi\sqrt{3}/3$$

$$23. \left(\frac{\pi}{4}, \sqrt{2}\right) \text{ and } \left(\frac{5\pi}{4}, -\sqrt{2}\right)$$

$$24. f(x) = x^2 g(x) \Rightarrow f'(x) = x^2 g'(x) + g(x)(2x) = x[xg'(x) + 2g(x)]$$

$$25. f(x) = [g(x)]^2 \Rightarrow f'(x) = 2[g(x)] \cdot g'(x) = 2g(x) g'(x)$$

$$26. f(x) = g(g(x)) \Rightarrow f'(x) = g'(g(x)) g'(x)$$

27. $f(x) = g(\sin x) \Rightarrow f'(x) = g'(\sin x) \cdot \cos x$

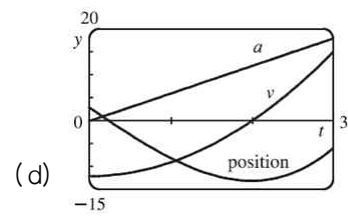
28. 생략

29.

(a) $v(t) = 3t^2 - 12, a(t) = 6t$

(b) $t > 2$ 일 때 위쪽, $0 \leq t < 2$ 일 때 아래쪽

(c) 23



30. $\frac{4}{3} \text{ cm}^2/\text{min}$

31.

(a) 1.01

(b) $-0.235 < x < 0.401$

32. $\frac{1}{32}$