

연습문제 해답

게시 일자 : 2018-05-14

10장

편도함수

10.1 다변수함수

01.

(a) 1

(b) \mathbb{R}^2

(c) $[-1, 1]$

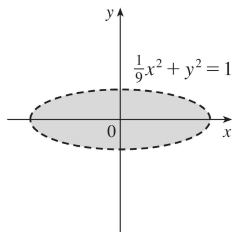
02.

(a) 3

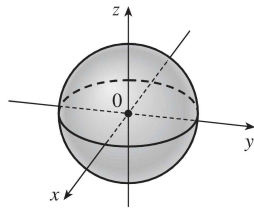
(b) $\{(x, y, z) \mid x^2 + y^2 + z^2 < 4, x \geq 0, y \geq 0, z \geq 0\}$,

반지름이 2이고 중심이 원점인 제1팔분공간 안에 놓이는 구의 내부

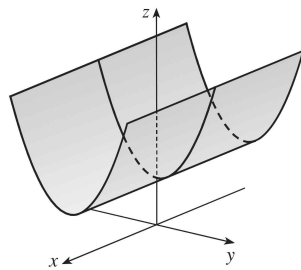
03. $\{(x, y) \mid \frac{1}{9}x^2 + y^2 < 1\}$



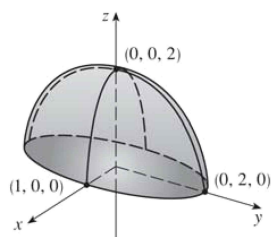
04. $\{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1\}$



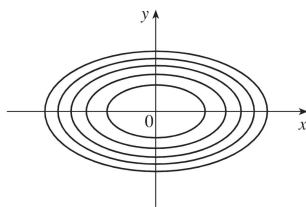
05.



06.



07.



08.

(a) C

(b) II

09.

(a) A

(b) IV

10.

(a) F

(b) I

11.

(a) E

(b) III

12.

(a) B

(b) VI

13.

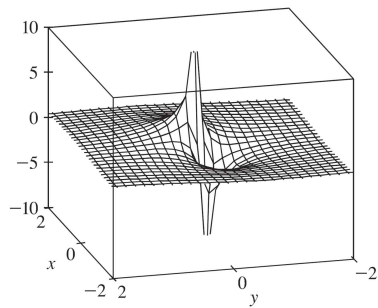
(a) D

(b) V

14. 평평한 평면족

15. 중심축이 x 축인 원기둥족 ($k>0$)

16.



x, y 가 커질수록 함숫값은 0으로 접근한다; (x, y) 가 원점으로 접근할수록 f 는 접근 방향에 따라 $\pm \infty$ 또는 0으로 접근한다.

10.2 극한과 연속성

01. 어느 것도 말할 수 없다. f 가 연속이면 $f(3, 1) = 6$ 이다.

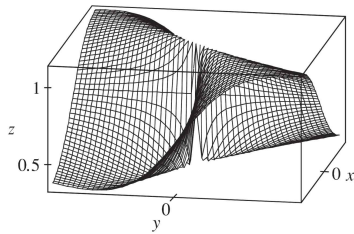
02. 1

03. 존재하지 않는다.

04. 0

05. 존재하지 않는다.

06.



07. $h(x, y) = (2x + 3y - 6)^2 + \sqrt{2x + 3y - 6}$; $\{(x, y) \mid 2x + 3y \geq 6\}$

08. $\{(x, y) \mid x^2 + y^2 \neq 1\}$

09. $\{(x, y) \mid x^2 + y^2 > 4\}$

10. $\{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1\}$

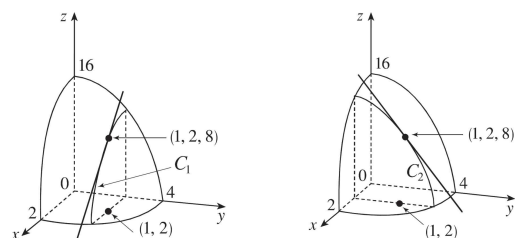
11. $\{(x, y) \mid (x, y) \neq (0, 0)\}$

12. 0

13. -1

10.3 편도함수

01. $f_x(1, 2) = -8 = C_1$ 의 기울기, $f_y(1, 2) = -4 = C_2$ 의 기울기;



02. $f_x(x, t) = -\pi e^{-t} \sin \pi x$, $f_t(x, t) = -e^{-t} \cos \pi t$

03. $f_x(x, y) = 1/y$, $f_y(x, y) = -x/y^2$

04. $f_x(x, y) = \frac{(ad-bc)y}{(cx+dy)^2}$, $f_y(x, y) = \frac{(bc-ad)x}{(cx+dy)^2}$

05. $F_x(x, y) = \cos(e^x)$, $F_y(x, y) = -\cos(e^y)$

06. $\frac{\partial w}{\partial x} = 1/(x+2y+3z)$, $\frac{\partial w}{\partial y} = 2/(x+2y+3z)$, $\frac{\partial w}{\partial z} = 3/(x+2y+3z)$

07. $\frac{\partial u}{\partial x} = y \sin^{-1}(yz)$, $\frac{\partial u}{\partial y} = x \sin^{-1}(yz) + xyz/\sqrt{1-y^2z^2}$, $\frac{\partial u}{\partial z} = xy^2/\sqrt{1-y^2z^2}$

08. $\frac{1}{5}$

09. $\frac{1}{4}$

10. $f_x(x, y) = y^2 - 3x^2y$, $f_y(x, y) = 2xy - x^3$

11. $\frac{\partial z}{\partial x} = -\frac{x}{3z}$, $\frac{\partial z}{\partial y} = -\frac{2y}{3z}$

12. $\frac{\partial z}{\partial x} = \frac{yz}{e^z - xy}$, $\frac{\partial z}{\partial y} = \frac{xz}{e^z - xy}$

13.

(a) $f'(x), g'(y)$

(b) $f'(x+y), f'(x+y)$

14. $f_{xx} = 6xy^5 + 24x^2y, f_{xy} = 15x^2y^4 + 8x^3 = f_{yx}, f_{yy} = 20x^3y^3$

15. $z_{xx} = -2x/(1+x^2)^2, z_{xy} = 0 = z_{yx}, z_{yy} = -2y/(1+y^2)^2$

16. $u = x^4y^3 - y^4 \Rightarrow u_x = 4x^3y^3, u_{xy} = 12x^3y^2, u_y = 3x^4y^2 - 4y^3, u_{yx} = 12x^3y^2$

따라서 $u_{xy} = u_{yx}$ 이다.

17. $(2x^2y^2z^5 + 6xyz^3 + 2z)e^{xyz^2}$

18. $\theta e^{r\theta} (2 \sin \theta + \theta \cos \theta + r\theta \sin \theta)$

19. 생략

20. 생략

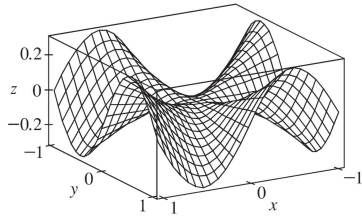
21. R^2/R_1^2

22. 생략

23. -2

24.

(a)

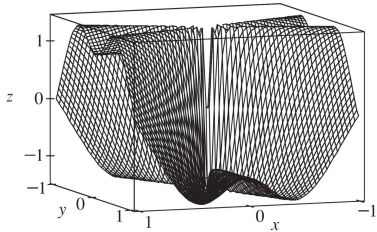


(b) $f_x(x, y) = \frac{x^4 y + 4x^2 y^3 - y^5}{(x^2 + y^2)^2}$, $f_y(x, y) = \frac{x^5 - 4x^3 y^2 - xy^4}{(x^2 + y^2)^2}$

(c) 0, 0

(d) 생략

(e) 아니다. f_{xy} 와 f_{yx} 가 연속이 아니기 때문이다.



10.4 접평면과 선형 근사

01. $z = -7x - 6y + 5$

02. $x + y + z = 0$

03. $6x + 4y - 23$

04. $1 - \pi y$

05. $\frac{3}{7}x + \frac{2}{7}y + \frac{6}{7}z; 6.9914$

06. $dm = 5p^4 q^3 dp + 3p^5 q^2 dq$

07. $dR = \beta^2 \cos \gamma d\alpha + 2\alpha\beta \cos \gamma d\beta - \alpha\beta^2 \sin \gamma d\gamma$

08. $\Delta z = 0.9225, dz = 0.9$

09. 5.4cm^2

10. 약 16cm^3

10.5 연쇄법칙

01. $(2x + y) \cos t + (2y + x) e^t$

02. $e^{y/z} [2t - (x/z) - (2xy/z^2)]$

03. $\partial z / \partial s = 2xy^3 \cos t + 3x^2y^2 \sin t, \partial z / \partial t = -2sxy^3 \sin t + 3sx^2y^2 \cos t$

04. $\frac{\partial z}{\partial s} = e^r \left(t \cos \theta - \frac{s}{\sqrt{s^2 + t^2}} \sin \theta \right), \frac{\partial z}{\partial t} = e^r \left(s \cos \theta - \frac{t}{\sqrt{s^2 + t^2}} \sin \theta \right)$

05. 1582, 3164, -700

06. $2\pi, -2\pi$

07. $-\frac{x}{3z}, -\frac{2y}{3z}$

08. $\frac{yz}{e^z - xy}, \frac{xz}{e^z - xy}$

09.

(a) $6\text{m}^3/\text{s}$

(b) $10\text{m}^2/\text{s}$

(c) $0\text{m}/\text{s}$

10. $\approx -0.27\text{L}/\text{s}$

11.

(a) $\partial z / \partial r = (\partial z / \partial x) \cos \theta + (\partial z / \partial y) \sin \theta, \partial z / \partial \theta = -(\partial z / \partial x) r \sin \theta + (\partial z / \partial y) r \cos \theta$

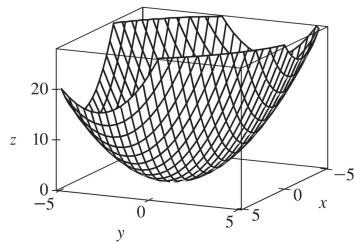
(b) 생략

12. 생략

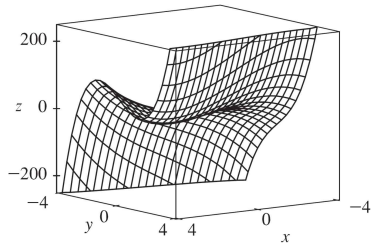
13. 생략

10.6 최댓값과 최솟값

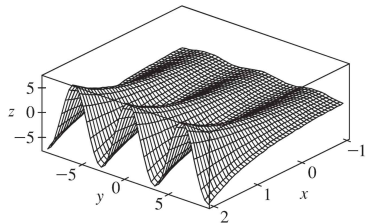
01. 극솟값 $f\left(\frac{1}{3}, -\frac{2}{3}\right) = -\frac{1}{3}$



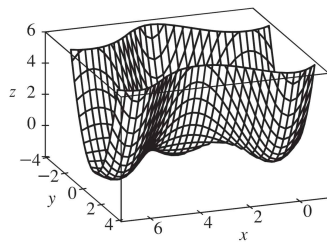
02. 극솟값 $f(2, 1) = -8$, 안장점 $(0, 0)$



03. 없다.



04. 극솟값 $f(0, 1) = f(\pi, -1) = f(2\pi, 1) = -1$, 안장점 $(\pi/2, 0)$, $(3\pi/2, 0)$



05. 극댓값 $f(0, 0) = 2$, 극솟값 $f(0, 2) = -2$, 안장점 $(\pm 1, 1)$

06. 극댓값 $f(\pi/3, \pi/3) = 3\sqrt{3}/2$, 극솟값 $f(5\pi/3, 5\pi/3) = -3\sqrt{3}/2$, 안장점 (π, π)

07. 최댓값 $f(0, \pm 2) = 4$, 최솟값 $f(1, 0) = -1$

08. 최댓값 $f(3, 0) = 83$, 최솟값 $f(1, 1) = 0$

09. $2/\sqrt{3}$

10. $(2, 1, \pm \sqrt{5})$

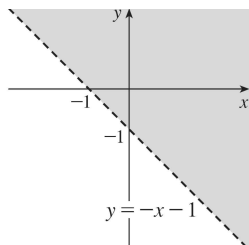
11. $\frac{4}{3}$

12. 밑변은 변의 길이가 40cm인 정사각형, 높이가 20cm

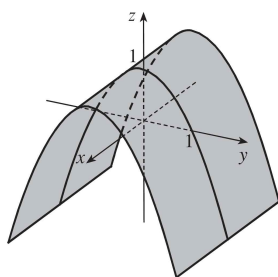
10장 복습문제

연습문제

01. $\{(x, y) \mid y > -x - 1\}$



02.



03. $\frac{2}{3}$

04. $F_\alpha = \frac{2\alpha^3}{\alpha^2 + \beta^2} + 2\alpha \ln(\alpha^2 + \beta^2), F_\beta = \frac{2\alpha^2\beta}{\alpha^2 + \beta^2}$

05. $S_u = \arctan(v\sqrt{w}), S_v = \frac{u\sqrt{w}}{1+v^2w}, S_w = \frac{uv}{2\sqrt{w}(1+v^2w)}$

06. $f_{xx} = 24x, f_{xy} = -2y = f_{yx}, f_{yy} = -2x$

07. $f_{xx} = k(k-1)x^{k-2}y^ly^m, f_{xy} = klx^{k-1}y^{l-1}z^m = f_{yx},$

$f_{xz} = kmx^{k-1}y^lz^{m-1} = f_{zx}, f_{yy} = l(l-1)x^ky^{l-2}z^m,$

$f_{yz} = lmx^ky^{l-1}z^{m-1} = f_{zy}, f_{zz} = m(m-1)x^ky^lz^{m-2}$

08. 생략

09.

(a) $z = 8x + 4y + 1$

(b) $\frac{x-1}{8} = \frac{y+2}{4} = \frac{z-1}{-1}$

10.

(a) $x + 2y + 5z = 0$

(b) $x - 2 = \frac{y+1}{2} = \frac{z}{5}$

11. $60x + \frac{24}{5}y + \frac{32}{5}z - 120; 38.656$

12. $2xy^3(1+6p) + 3x^2y^2(pe^p + e^p) + 4z^3(p \cos p + \sin p)$

13. 생략

14. 생략

15. 최댓값: $f(1, 2) = 4$, 최솟값: $f(2, 4) = -64$

16. $(\pm 3^{-1/4}, 3^{-1/4}\sqrt{2}, \pm 3^{1/4}), (\pm 3^{-1/4}, -3^{-1/4}\sqrt{2}, \pm 3^{1/4})$

17. $a = \frac{2\sqrt{3}-3}{3}P$, $b = \frac{3-\sqrt{3}}{6}P$, $c = (2-\sqrt{3})P$

