MSE, 기초 공업수학

[연습문제 정답 이용 안내]

- 본 연습문제 정답의 저작권은 **마인숙**과 **한빛아카데미(주)**에 있습니다.
- 이 자료를 무단으로 전제하거나 배포할 경우 저작권법 136조에 의거하여 최고 5년 이하의 징역 또는 5천만원 이하의 벌금에 처할 수 있고 이를 병과(倂科)할 수도 있습니다.

Chapter 01 연습문제 정답

[1.1 도함수]

1. $f'(x) = 20(x+x^3)^{19}(1+3x^2)$

2. $\dfrac{-t^2-4t-2}{(t^2-2)^2}$

3. $g'(x) = 6x^2(x^3+1)(x^2-1)^4 + 8x(x^3+1)^2(x^2-1)^3$

4. $\dfrac{4t^5-5t^4+1}{(t-1)^2}$

5. $\dfrac{d}{dt}\{f(x^2+1)\} = 2xf'(x^2+1)\dfrac{dx}{dt}$

6. $-\dfrac{2}{(x-1)^2}f'\left(\dfrac{x+1}{x-1}\right)\dfrac{dx}{dt}$

7. $\dfrac{d}{dt}\left\{\dfrac{f(x)+1}{f(x)-1}\right\} = \dfrac{-2f'(x)}{(f(x)-1)^2}\dfrac{dx}{dt}$

8. $\dfrac{dy}{dx} = -\dfrac{2x+y^2}{y(2x+9y)}$

9. $\dfrac{dy}{dx} = \dfrac{1}{2y-2}$

10. $4(2x+1)\big(3(2x+1)^4+2\big)$

11. $\dfrac{dy}{dx} = -\dfrac{2x+y}{x+2y}$

[1.2 여러 함수의 미분법]

12. $-3x^2\sin(x^3-1)$

13. $y'=2\sec^2 x\tan x$

14. $\dfrac{dy}{dx}=\dfrac{1}{2y+\sec^2 y}$

15. $\dfrac{dy}{dx}=\dfrac{\sin x+y^2}{\cos y-2xy}$

16. $-\dfrac{2}{x\sqrt{x^4-1}}$

17. $\dfrac{dy}{dx}=-\dfrac{\cos x}{\sqrt{1-\sin^2 x}}$

18. $\dfrac{x}{(x^2-1)\sqrt{x^2-2}}$

19. $\dfrac{dy}{dx}=\dfrac{1}{2(1+x)\sqrt{x}}$

20. $-\tan x$

21. $\dfrac{dy}{dx}=\dfrac{\sin^3 x+\cos^2 x}{\cos x\sin^2 x-\cos^3 x\sin x}$

22. $2x\ln x+x$

23. $\dfrac{dy}{dx}=\dfrac{1}{2x}$

24. $3^x\ln 3\sin x+3^x\cos x$

25. $\dfrac{dy}{dx}=-\dfrac{3e^{3x}}{2\sqrt{1-e^{3x}}}$

26. $e^x\left(\ln x+\dfrac{1}{x}\right)$

27. $\dfrac{dy}{dx}=x^{\cos x}\left(-\sin x\ln x+\dfrac{1}{x}\cos x\right)$

28. $6x^2\sinh(2x^3)$

29. $\frac{dy}{dx} = -3x^2\operatorname{sech}^2(1-x^3)$

30. $\frac{6x}{\sqrt{1+9x^4}}$

31. $\frac{dy}{dx} = 2\frac{\cosh^{-1}x}{\sqrt{x^2-1}}$

[1.3 편도함수]

32. $\frac{9z^2}{x^2-2y+3z^3}$

33. $\frac{\partial f}{\partial z}(1,2,3) = -\frac{6}{169}$

34. $4x\sin(x-2y)+2x^2\cos(x-2y)$

35. $\frac{\partial z}{\partial x} = -\frac{2x+2yz}{2xy+2z}$

Chapter 02 연습문제 정답

[2.1 부정적분과 정적분]

1. $\int x^{\frac{1}{3}}\,dx = \frac{3}{4}x^{\frac{4}{3}} + C$

2. $\frac{1}{2}x^2 + \frac{4}{3}x^{\frac{3}{2}} + x + C$

3. $\int x(1+\sqrt{x})\,dx = \frac{1}{2}x^2 + \frac{2}{5}x^{\frac{5}{2}} + C$

4. $2\sqrt{x} + C$

5. 36

6. $\frac{13}{12}$

7. 13

8. $\frac{38}{45}$

9. $\frac{16}{5} - \frac{6}{5}2^{\frac{5}{3}}$

10. $\frac{41}{35}$

[2.2 여러 함수의 적분법]

11. $\int \tan 2x\,dx = -\frac{1}{2}\ln|\cos 2x| + C$

12. $-\frac{1}{2}\cos t^2 + C$

13. $\int \tan^2 x\,dx = \tan x - x + C$

14. $\frac{1}{2}\tan^2 x + \ln|\cos x| + C$

15. $\int \sin^3\theta \, d\theta = -\cos\theta + \frac{1}{3}cos^3\theta + C$

16. $\tan\theta - \sec\theta + C$

17. $\int \frac{\cos^2 x}{1-\sin x}\, dx = x - \cos x + C$

18. $\frac{1}{2}\sin^4 x + C$

19. $\int \frac{e^x}{1-e^x}\, dx = -\ln|1-e^x| + C$

20. $\ln|1-\cos x| + C$

21. $\int \frac{1}{t(1-\ln t)}\, dt = -\ln|1-\ln t| + C$

22. $\ln(e^x + 1) + C$

23. $\int \frac{e^{2x} + e^{-2x}}{3}\, dx = \frac{1}{6}(e^{2x} - e^{-2x}) + C$

24. $\frac{1}{2}(\ln(\ln x))^2 + C$

25. $\int x^2 e^{x^3}\, dx = \frac{1}{3}e^{x^3} + C$

26. $\frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x} + 2x + C$

27. $\int \frac{1}{x\sqrt{9-4x^2}}\, dx = -\frac{1}{3}\ln\left|\frac{3}{2x}\left(1 + \sqrt{1-\frac{4x^2}{9}}\right)\right| + C$

28. $\frac{1}{3}\tan^{-1}\left(\frac{x+1}{3}\right) + C$

29. $\int \frac{1}{\sqrt{4-x^2}}\, dx = \sin^{-1}\frac{x}{2} + C$

30. $\frac{1}{6}\sec^{-1}\frac{2}{3}x+C$

31. $\int\frac{1}{t\sqrt{t+1}}\,dt=\ln|\sqrt{t+1}-1|-\ln|\sqrt{t+1}+1|+C$

32. $\frac{1}{2}\sec^{-1}\frac{x-1}{2}+C$

33. $\int x\sqrt{x+1}\,dx=2\left(\frac{1}{5}\sqrt{(x+1)^5}-\frac{1}{3}\sqrt{(x+1)^3}\right)+C$

34. $\sqrt{x^2-1}-\tan^{-1}\sqrt{x^2-1}+C$

[2.3 부분분수에 의한 유리함수의 적분법]

35. $-\frac{1}{2}\ln|x|+\frac{1}{3}\ln|x+1|+\frac{1}{6}\ln|x-2|+C$

36. $\frac{2}{9}\ln\left|\frac{x-1}{x+2}\right|-\frac{1}{3}\frac{1}{x+2}+C$

37. $\ln|x-1|-\frac{2}{x-1}-\frac{1}{2}\frac{1}{(x-1)^2}+C$

38. $\ln|x|-\frac{1}{2}\ln(x^2-x+1)+\frac{1}{\sqrt{3}}\tan^{-1}\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}+C$

39. $4\ln|x|-2\ln(x^2+1)+\frac{2}{x^2+1}+C$

[2.4 부분적분법]

40. $\int e^x\cos x\,dx=\frac{1}{2}e^x(\sin x+\cos x)+C$

41. $x\tan^{-1}x-\frac{1}{2}\ln(1+x^2)+C$

42. $\sin x-\frac{1}{3}\sin^3 x+C$

43. $x\sin^{-1}x+\sqrt{1-x^2}+C$

44. $x(\ln x)^2 - 2x\ln x + 2x + C$

45. $(x^2 - x)\ln(x-1) - \frac{1}{2}x^2 + C$

46. $\frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2\ln x + \frac{1}{4}x^2 + C$

47. $\frac{x}{1-x}\ln x + \ln|x-1| + C$

Chapter 03 연습문제 정답

[3.1 미분방정식의 소개]

1. 2계 1차

2. 2계 2차

3. 1계 1차

4. 2계 1차

5. 3계 1차

6. 3계 1차

7. 선형

8. 비선형

9. 선형

10. 비선형

11. 비선형

12. 선형

13. $c_1 = 1,\ c_2 = 1$

14. $c_1 = 0,\ c_2 = 1$

15. $c_1 = 0,\ c_2 = -1$

16. $c_1 = 0,\ c_2 = 2$

17. 존재하지 않는다.

18. $x^3 + (y+1)^3 = C \quad (C = 3C')$

[3.2 변수분리형 미분방정식]

19. $(x+1)^2+(y+1)^2+2\ln|(x+1)(y-1)|=C$

20. $y=C\frac{x}{x-1}$

21. $yx^2=ce^y$

22. $\frac{1}{x^2}-\frac{1}{y^2}=-2c'=c$

23. $\sqrt{2}$

24. $\sqrt{2}$

25. $\frac{\pi}{4}$

26. 999.99

27. $10\times e^{-5}$

28. 동차 2차

29. 동차 0차

30. 동차 $\frac{3}{2}$차

31. 동차 아님

32. 동차 3차

33. 동차 0차

34. 동차 -1차

35. 동차 -2차

36. $\ln|x|+\frac{\sqrt{41}+1}{2\sqrt{41}}\ln|\frac{2y}{x}+7-\sqrt{41}|+\frac{\sqrt{41}-1}{2\sqrt{41}}\ln|\frac{2y}{x}+7+\sqrt{41}|=c$

37. $x^3-2y^3=cx$ (c : 임의 상수)

38. $c\sin^3\frac{y}{x}$

39. $y=c(\frac{y}{x}-1)$ (c : 임의 상수)

40. $x\ln|\sqrt{x^2+y^2}-x|=cx-\sqrt{x^2+y^2}$

41. $x^2+4xy+3y^2=c$ (c : 임의 상수)

42. $\ln|x|=-e^{\frac{y}{x}}+1$

43. $1+\cos\frac{x}{y}=\frac{\pi}{2}y\sin\frac{x}{y}$

44. $y\ln\left(\frac{x}{y}\right)=1$

45. $x^2(1+2\sqrt{\frac{y}{x}})=e^{2\sqrt{\frac{y}{x}}}$

46. $x+2y+\ln|x+y|=c$

47. $\frac{5}{2}(x-y)-\ln|15x+10y-1|=c$

48. $\frac{1+\sqrt{3}(2x+3y)}{1-\sqrt{3}(2x+3y)}=ce^{4\sqrt{3}x}$ (c는 임의 상수)

49. $4(x+2\sin y)+9\ln|4x-8\sin y+3|=c$

50. $1-\cos(x+y)=(x+c)\sin(x+y)$

51. $2\sqrt{2x+y+1}-\ln(2\sqrt{2x+y+1}+1)=2y+c$

52. $e^{x+y+2}=\frac{3ce^{3x}}{1-ce^{3x}}$

[3.3 완전 미분방정식]

53. 일반해 : $ye^{x^2}+x^2=c$

54. 일반해 : $e^{3x}y+x^2=c$

55. 일반해 : $\frac{1}{2}x^4+3xy+\frac{1}{3}y^3-y=c$ 또는 $3x^4+18xy+2y^3-6y=c$

56. 일반해 : $e^{xy^2}+\frac{2}{3}x^3-y^3=c$

57. 일반해 : $-\rho+\rho e^{2\theta}=c$

58. 일반해 : $x^4y^3+\ln x+\ln y=c$

59. 일반해 : $\frac{1}{3}(x^2+y^2)^{\frac{3}{2}}-xy=c$

60. 일반해 : $r\csc\theta+\ln\cos\theta=c$

61. 일반해 : $xy^2-\ln x+\ln(x+y)+x+y^2=c$

62. 일반해 : $e^{x^2y}+e^{xy^2}+2x+y^2=c$

63. 일반해 : $\frac{1}{4}x^4+\frac{1}{2}x^2y^2+\frac{1}{3}x^3+\frac{1}{2}x^2=c$ 또는 $3x^4+6x^2y^2+4x^3+6x^2=c$

64. 일반해 : $x^2e^y+\frac{x^2}{y}+\frac{x}{y^3}=c$

65. 일반해 : $e^{x^2}(\frac{1}{2}y^4+x^2y^2+2xy)=c$

66. 일반해 : $\frac{y}{x}+\frac{1}{x}+x=c$ or $y+1+x^2=cx$

67. 일반해 : $x^2y-x^3=c$

68. 일반해 : $\ln x+\frac{y^2}{x}=c$

69. 일반해 : $e^x \sin y + y^2 = c$

70. 일반해 : $x^2 e^x \sin y = c$

[3.4 선형 미분방정식]

71. $y\cos^2 x = \int \sin 2x\, dx = -\frac{1}{2}cos2x + c$

72. $2yx^3 = 2ce^{\frac{1}{x^2}} + 1$

73. $y = \sin 2x(x - \frac{1}{2}\ln|\csc 2x + \cot 2x| + c)$

74. $r = \sin^2\theta(2\ln|\sin\theta| + c)$

75. $y = c(1 + e^x)$

76. $y = \frac{1}{1+x^3}(\frac{1}{3}x^3 + c)$

77. $i = -\frac{1}{2}\cos 2t + \frac{1}{4}\frac{\sin 2t}{t} + \frac{c}{t}$

78. $y = \frac{1}{x}e^x(-\frac{1}{2}\cos 2x + c)$

79. $x = \frac{4}{7}y^3 + cy^{-4}$

80. $(y^2 - 2y + 2)e^y + c$

81. $y = \begin{cases} \frac{1}{x^2} & 0 < x \leqq 2 \\ \frac{1}{3}x - \frac{5}{3}\frac{1}{x^2} & x > 2 \end{cases}$

82. $y = \begin{cases} \frac{1}{3} + \frac{5}{3}e^{-x^3} & 0 \leqq x < 1 \\ (\frac{1}{3}e + \frac{5}{3})e^{-x^3} & x \geqq 1 \end{cases}$

83. $y=\begin{cases}2+e^{-x^2} & 0 \leqq x<1 \\ 4x^2\ln x+(e^{-1}-2)x^2 & x \geqq 1\end{cases}$

84. $y^3(e^{3x^2}+c)=2e^{3x^2} \qquad (c=2c_1)$

85. $y^2(-2x+ce^{2x})=1$

86. $e^x=(-e^x\sin x+c)y$

87. $y\left(-\frac{1}{4}x-\frac{1}{2}x\ln x+\frac{c}{x}\right)=1$

Chapter 04 연습문제 정답

[4.1 1계 선형모형]

1. $x = 6.727171\,x_0$

2. $x = 3\times 10^5 e^{0.099021t}$

3. $y = e^{\frac{1}{5}(t-\cos t+1)}\,y_0$

4. $T = 290 + 80e^{-0.028768t}$

5. $241920(J)$

6. $A = A_0 e^{-0.000433t}$

7. $0.574(g)$

8. $x(3600) = 20e^{-3\times 10^{-4}\times 3600} \fallingdotseq 6.79(kg)$

9. $18.08\,(kg)$

10. 소금 양 $484(kg)$

    소금의 농도 : $\frac{484}{500}(kg/l) = 0.968$

11. 0.06%

12. $i = 55 + ce^{-4t}$

13. $i(t) = 2(\sin t + 2\cos t) + ce^{-2t}$

14. 2.217164

15. $v = \frac{18}{49}(-\frac{1}{2}\cos 2t + \frac{49}{20}\sin 2t) + ce^{-\frac{49}{5}t}$

16. $512.69(s)$

17. $t = 38.88$

18. $\frac{10^6}{3^9}\left(3125 \ln \frac{625}{463} - 810\right) \approx 6482\,(\text{초})$

19. 5.7개월

20. $\frac{1}{30}x_1 - \frac{2}{100+t}x_2 + q_2$

21. $\frac{dx_1}{dt} = \left(1.5 + \frac{1.5}{20-1.5t}x_2\right) - \left(\frac{x_1}{30} \cdot 3\right)$

$\frac{dx_2}{dt} = \left(3 + \frac{x_1}{30} \cdot 3\right) - \left(\frac{x_2}{20-1.5t} \cdot 5.5\right)$

22 $\begin{cases} -4i_3' + 6i_2' + 4i_2 = 0 \\ i_2' + i_3' + 4i_3 = 12 \end{cases}$

23. $10i_2 + 10i_3 + 4i_3' = 4$

$10i_2' + 10i_3' + 100i_2 = 0$

[4.2 비선형모형]

24. (a) $c_1 = \frac{y_0}{1000 - y_0}$, $y = \frac{1000\,y_0}{(1000 - y_0)e^{-1000\alpha t} + y_0}$

(b) 0명

25. $P(t) = \frac{2\times10^6\ [-3(2\times10^4 - 100)]^t}{(2\times10^4 - 100)(300 - 2\times10^4)^t + 100[-3(2\times10^4 - 100)]^t}$

26. (a) 10^5 (명) (b) 35개월

27. (a) $x = \frac{bce^{\alpha(a-b)t} - a}{ce^{\alpha(a-b)t} - 1}$ => $\begin{cases} ①a > b : t \to \infty \text{일 때 } x \to b \\ ②a < b : t \to \infty \text{일 때 } x \to a \end{cases}$

(b) $x = a - \frac{1}{\alpha t + c_1}$ 그리고 $t \to \infty : x \to a$

28. (a) 27.41 (b) $\frac{200}{3}$ (c) A의 양은 $\frac{100}{3}\,(g)$, B의 양은 $0\,(g)$

[4.3 해의 존재성과 유일성]

29. 생략

30. 판단할 수 없다.

31. $y < -3$이거나 $y > 3$

32. x는 모든 실수이고 $y \neq 4$인 모든 실수이다.

33. $y \neq x$인 모든 점

34. ① $y(0) = k \neq 0$이거나 $y(1) = k \neq 0$이면
해가 존재하지 않음
② $y(0) = 0$이거나 $y(1) = 0$이면
무수히 많은 해가 존재
③ 임의의 y_0에 대해, $y(x_0) = y_0$, $x_0 \neq 0$
이고 $x_0 \neq 4$이면 유일한 해 존재

Chapter 05 연습문제 정답

[5.1 선형미분방정식의 기본이론]

1. $2x+x^2$

2. $(4x+4)e^x$

3. e^{2x}

4. $a\cos ax+a\sin ax$

5. $2D^2-D-6$

6. $x^2D^2+2xD-2$

7. D^2-4

8. $x^2D^2+D-x^2-x$

9. $(D+3)(D-3)$

10. $(2D+1)(D+1)$

11. $D(D+2)$

12. $D(D-1)(D-2)$

13. $(D-2)(D-6)$

14. $(D^2+2+\sqrt{5}\,D)\,(D^2+2-\sqrt{5}\,D)$

15. $D^3(1+3x+2x^2)=0$

16. $(D^2-2D)y=0$

17. 생략

18. $\cos x\neq 0$인 구간 I에서 연속

[5.2 선형미분방정식의 성질]

19. 일차독립

20. 일차독립

21. 일차종속

22. 일차독립

23. 일차독립

24. 일차독립

25. 일차독립

26. 일반해 $y = c_1 e^{3x} + c_2 e^{4x}$

27. 일반해 $y = c_1 x^4 + c_2 x^{-\frac{1}{2}}$

28. 일반해 $y = c_1 \cos(2\ln x) + c_2 \sin(2\ln x)$

29. 일반해 $y = c_1 e^x \cos 2x + c_2 e^x \sin 2x = e^x (c_1 \cos 2x + c_2 \sin 2x)$

30. $= e^x((-x+1)e^{-x} - e^{-x}) = -x$

31. x^3

32. xe^x

33. $\frac{2}{3} x^{\frac{1}{2}}$

34. $\frac{1}{2} \sin x^2$

35. $\frac{\sin x}{x}$

[5.3 상수계수를 갖는 선형 제차 미분방정식]

36. $y = c_1 e^{-x} + c_2 e^{\frac{1}{2}x}$

37. $y = c_1 + c_2 e^{\frac{1}{2}x} + c_3 x e^{\frac{1}{2}x}$

38. $y = c_1 e^{-x} + e^{\frac{1}{2}x}\left(c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x\right)$

39. $y = c_1 \cos 3x + c_2 \sin 3x + c_3 e^{\sqrt{2}\,x} + c_4 e^{-\sqrt{2}\,x}$

40. $y = c_1 + c_2 e^{\sqrt{3}\,x} + c_3 e^{-\sqrt{3}\,x} + c_4 \cos\sqrt{3}\,x + c_5 \sin\sqrt{3}\,x$

41. $y = c_1 \cos x + c_2 \sin x + c_3 x \cos x + c_4 x \sin x$

42. $y = c_1 + c_2 x + e^{\frac{1}{2}x}\left(c_3 \cos\frac{\sqrt{3}}{2}x + c_4 \sin\frac{\sqrt{3}}{2}x\right)$

43. $y = \cos 2x - \frac{1}{2}\sin 2x$

44. $y = \frac{3}{4}e^{x} + \frac{1}{4}e^{5x}$

45. $y = \frac{23}{18} - \frac{5}{18}e^{6x} + \frac{2}{3}xe^{6x}$

46. $y = x + x^2 + x^3$

47. $y = (1-x)e^{-5x}$

48. $y = \cos 3x + \sin 3x$

49. $y''' - 3y'' + 4y' - 2y = 0$

50. $y'' + 4y = 0$

51. $y''' - 4y'' + y' + 6y = 0$

52. $y'' - 9y = 0$

53. $y''' - y'' = 0$

54. $y'' + 4y = 0$

[5.4 미정계수법]

55. $y = c_1 e^{-3x} + c_2 e^{x} + \frac{1}{5} e^{2x}$

56. $y = c_1 e^{-3x} + c_2 e^{x} - \frac{1}{4} x e^{-3x}$

57. $y = e^{x}(c_1 \cos 2x + c_2 \sin 2x) + \frac{2}{17}\cos 2x - \frac{8}{17}\sin 2x$

58. $y = e^{x}(c_1 \cos 2x + c_2 \sin 2x) - \frac{3}{4} x e^{x} \cos 2x$

59. $y = \frac{3}{4} e^{-2x} + \frac{3}{4} e^{2x} - \frac{1}{2}$

60. $y = -2 + 4e^{-x} + x e^{-x} + 5x - (\frac{1}{6}x^3 + \frac{1}{2}x^2) e^{-x}$

61. $y = e^{-2x}(-\frac{1}{32}\cos 2x + \frac{1}{32}\sin 2x) + \frac{1}{8}x^2 - \frac{1}{8}x + \frac{1}{32}$

62. $y = 4 - 3x + x^2 - 4e^{-x} - \frac{1}{6}x^3 + \frac{1}{24}x^4 - x e^{-x}$

63. $y_P = Ax\cos 2x + Bx\sin 2x + Cx^2\cos 2x + Ex^2 \sin 2x$

[5.5 매개변수 변화법]

64. $y = c_1 e^{2x} + c_2 e^{-2x} + \frac{1}{4} e^{2x} \int \frac{1}{x} e^{-4x} dx - \frac{1}{4} e^{-2x} \ln|x|$

65. $y = c_1 \cos x + c_2 \sin x - \cos x \ln|\sec x + \tan x|$

66. $y = c_1 e^{-x} + c_2 e^{-2x} + \frac{1}{12} e^{2x} - \frac{1}{6} e^{x} + e^{-x} - (e^{-x} + e^{-2x}) \ln(1 + e^{x}) + \frac{1}{2}$

67. $y = c_1 e^{-x} + c_2 e^{-2x} - e^{-2x}\cos e^{x}$

68. $y = e^{-x}(c_1\cos x + c_2\sin x) + e^{-x}(\cos x \cdot \ln|\cos x| + x\sin x)$

69. $y = c_1 e^{5x} + c_2 x e^{5x} + x e^{5x}(-1+\ln|x|) = c_1 e^{5x} + c_2 x e^{5x} + x e^{5x}\ln|x|$

70. $y = e^{-x}(c_1\cos 3x + c_2\sin 3x) - \frac{1}{9}(e^{-x}\cos 3x)\ln|\sec 3x + \tan 3x|$

71. $y = c_1 e^{-\frac{1}{2}x} + c_2 x e^{-\frac{1}{2}x} + \frac{1}{12}(1-x^2)^{\frac{3}{2}} e^{-\frac{1}{2}x} + \frac{1}{8} x e^{-\frac{1}{2}x}\left[\sin^{-1}x + x\sqrt{1-x^2}\right]$

72. $y = c_1 x + c_2 x\ln x + \frac{1}{4x}$

73. $y = c_1 x^{-\frac{1}{2}} + c_2 x - \frac{8}{9}x + \frac{2}{5}x^2 + \frac{4}{3}x\ln|x|$

[5.6 코시-오일러 방정식]

74. $y = c_1 x^{-3} + c_2 x^2 - \frac{1}{125}x^2(5\ln x - 1) + \frac{1}{10}x^2(\ln x)^2$

75. $y = c_1 x^2 + c_2 x^2\ln x + \frac{1}{4}\ln x + \frac{1}{4}$

76. $y = c_1 x^{-2} + c_2 x^2 - \frac{1}{2}\cos(\ln x) + \frac{3}{20}\sin(\ln x)$

77. $y = c_1 + c_2 x^{-3} + \frac{1}{28}x^4$

78. $y = c_1 x + c_2 x\ln x + x^2$

79. $y = c_1 x + c_2 x^2 - x e^{x} + x^2\int \frac{1}{x} e^{x} dx$

80. $y_c = c_1 x + c_2 x\ln x + c_3 x(\ln x)^2$

81. $y = c_1 x^{-2} + c_2 x^2 - \frac{1}{4x} + \frac{1}{4x^2}\ln|x+1| + \frac{1}{4}x^2\ln|x| + \frac{x}{4} - \frac{1}{8} - \frac{1}{4}x^2\ln|x+1|$

82. $a\dfrac{d^2y}{dt^2}+(b-a)\dfrac{dy}{dt}+cy=0$

83. $y=c_1x+c_2x^2+(\ln x-1)x^2+2$

Chapter 06 연습문제 정답

[6.1 고계 선형모형]

1. $x=5\sin(2t+\phi)$

   단 $\sin\phi=\frac{3}{5},\ \cos\phi=\frac{4}{5},\ \tan\phi=\frac{3}{4}$

   즉, $\Phi=36.8698^\circ\ (0.6435\,rad)$

2. $x=3\sin(t+\phi)$

   단, $\sin\phi=-\frac{2}{3},\ \cos\phi=\frac{\sqrt{5}}{3},\ \tan\phi=-\frac{2}{\sqrt{5}}$

   즉, $\Phi=-41.8104^\circ\ (-0.72973\,\mathrm{rad})$

3. $x=2\sqrt{2}\sin(3t+\frac{3}{4}\pi)$

   단 $\sin\phi=\frac{1}{\sqrt{2}},\ \cos\phi=\frac{-1}{\sqrt{2}}$

   즉, $\Phi=\frac{3}{4}\pi\ (rad)=135^\circ$

4. $x=\sqrt{5}\sin(\pi t+\phi)$

   단 $\sin\phi=\frac{-1}{\sqrt{5}},\ \cos\phi=-\frac{2}{\sqrt{5}},\ \tan\phi=\frac{1}{2}$

   즉, $\Phi=26.5623^\circ\ (0.4636\,\mathrm{rad})$

5. $x=3\cos\frac{7}{\sqrt{30}}t$

6. $x=\frac{16}{7}\sin(\frac{7}{\sqrt{15}}t+\Phi)$

   단, $\sin\Phi=-\frac{7}{8},\ \cos\Phi=\frac{\sqrt{15}}{8},\ \tan\Phi=-\frac{7}{\sqrt{15}}$

   즉, $\Phi=-61.045^\circ\ (-1.0654\,\mathrm{rad})$

7. $x=\frac{5}{7}\sin14t$

8. $x=e^{-10t}(2\cos4\sqrt{6}\,t+\frac{5}{\sqrt{6}}\sin4\sqrt{6}\,t)$

[6.2 고계 연립 선형 미분방정식의 모형]

9. $x_1 = c_1\cos 2t + c_2\sin 2t$

$x_2 = -c_1\sin 2t + c_2\cos 2t$

10. $x_1 = e^{\frac{5}{2}t}(c_1\cos\frac{\sqrt{7}}{2}t + c_2\sin\frac{\sqrt{7}}{2}t)$

$x_2 = e^{\frac{5}{2}t}(-(\frac{3}{4}c_1 + \frac{\sqrt{7}}{4}c_2)\cos\frac{\sqrt{7}}{2}t + (-\frac{3}{4}c_2 + \frac{\sqrt{7}}{4}c_1)\sin\frac{\sqrt{7}}{2}t)$

11. $x_1 = -\frac{13}{2}e^{2t} - \frac{3-2\sqrt{2}}{1-\sqrt{2}}c_2e^{(1-\sqrt{2})t} - \frac{3+2\sqrt{2}}{1+\sqrt{2}}c_3e^{(1+\sqrt{2})t}$

$x_2 = c_1 + c_2e^{(1-\sqrt{2})t} + c_3e^{(1+\sqrt{2})t} + \frac{7}{2}e^{2t}$

12. $x = 2c_1e^t + 2c_2(t-1)e^t - c_3e^{-2t}$

$y = c_1e^t + c_2te^t + c_3e^{-2t}$

$z = (c_1 - 2c_2)e^t + c_2(t-1)e^t + c_3e^{-2t}$

13. $q_1' - q_2' - q_3^3 = 0$

$q_2' - q_4' - q_5^4 = 0$

$10(q_3' + q_4' + q_5') + 5q_3'' = 6$

$10q_3' + 40q_4' + 40q_5'' + 10q_5 = 6$

$10q_3' + 55q_4' + 40q_5 + \frac{5}{2}q_4 = 6$

14. $\frac{3}{2}c_1e^{-\frac{1}{10}t} - c_2e^{-\frac{3}{5}t},\ c_1e^{-\frac{1}{10}t} + c_2e^{-\frac{3}{5}t}$

15. $(D+0.1)x_1 - 1.5x_2 = 1.5$

$0.1x_1 - (D + \frac{3}{20+t})x_2 = 3$

16. $86.4e^{-2.16} \fallingdotseq 9.9641(kg)$

Chapter 07 연습문제 정답

[7.1 라플라스 변환]

1. $e^{-s}(\frac{1}{s}+\frac{1}{s^2})$

2. $F(s)=\frac{s}{s^2+1}(e^{-\pi s}+1)$

3. $-\frac{k}{s}(e^{-sa}-1)$

4. $F(s)=\frac{1}{s}(e^{-3s}-2e^{-s}+1)$

5. $\frac{s}{s^2+\pi^2}$

6. $\frac{w\cos\theta+s\sin\theta}{s^2+w^2}$

7. $L\{\sin^2 x\}\frac{1}{2}(\frac{1}{s}-\frac{s}{s^2+4})$

8. $e^{-b}\frac{1}{s-a}$

9. $\frac{1}{s^2-2s+2}$

10. $\frac{s+1}{s^2+2s+2}$

11. 생략

12. $\frac{1}{s^{\alpha+1}}\Gamma(\alpha+1),\quad \alpha>-1$

13. $\frac{\sqrt{\pi}}{\sqrt{s}}$

14. $\frac{1}{2}\frac{\sqrt{\pi}}{s^{\frac{3}{2}}}$

15. $\frac{3\sqrt{\pi}}{4\,s^{\frac{5}{2}}}$

16. $\frac{1}{5!}t^5$

17. $\frac{1}{2}e^{\frac{\sqrt{2}}{2}t}$

18. $2\cos 2t$

19. $\frac{1}{2}\sin 2t$

20. $\cos\frac{1}{2}t$

21. $\sin\frac{1}{2}t$

22. $\cosh 5t - \frac{6}{5}\sinh 5t$

23. $\cos 6t - \frac{6}{5}\sin 5t$

24. $-e^{-t} + e^{4t}$

25. $\frac{1}{5}e^{4t} + \frac{1}{5}e^{-t}$

26. $1 + 3\cos 3t - \sin 3t$

27. $\frac{2}{13}e^{2t} - \frac{2}{13}\cos 3t + \frac{3}{13}\sin 3t$

28. $y(t) = \frac{4}{3}\cos t + \sin t - \frac{1}{3}cos2t$

29. $y(t)=2e^{t}-e^{2t}$

30. $y(t)=\cos t+\frac{2}{3}\sin t-\frac{1}{3}\sin 2t$

31. $y(t)=\frac{1}{2}(e^{t}-e^{-t})$

[7.2 라플라스 변환의 성질]

32. $\frac{1}{(s-2)^2}$

33. $\frac{2}{(s+2)^3}$

34. $\frac{2}{(s-2)^3}+2\frac{1}{(s-2)^2}+\frac{1}{s-2}$

35. $\frac{2}{(s-2)^2+4}$

36. $\frac{s-2}{(s-2)^2+4}$

37. $\frac{1}{2}t^2e^{-t}$

38. $\frac{1}{6}t^3e^{t}$

39. $(1+3t)e^{-2t}$

40. $(2\cos\sqrt{3}\,t-\frac{2}{\sqrt{3}}\sin\sqrt{3}\,t)e^{t}$

41. $(7\cos t+6\sin t)e^{-t}$

42. $(t-t^2+\frac{2}{3}t^3)e^{t}$

43. $(1+3t\,)\,e^{-2t}$

44. $(2\cos\sqrt{3}\,t-\frac{2}{\sqrt{3}}\sin\sqrt{3}\,t)e^{t}$

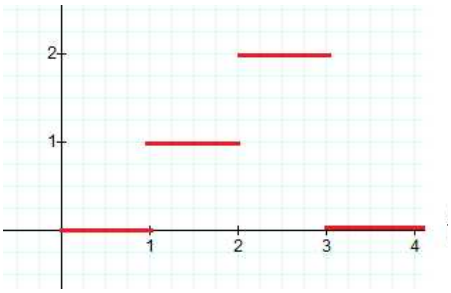
45. $y(t)=(\frac{7}{5}\cos t+\frac{6}{5}\sin t)e^{-t}-\frac{2}{5}\cos t+\frac{1}{5}\sin t$

46. $y(t)=(-\cos t+\sin t)e^{-t}+e^{-t}$

47. $y(t)=-\frac{3}{4}e^{t}-\frac{1}{2}t^{2}e^{t}+\frac{1}{12}e^{-t}+\frac{2}{3}e^{2t}$

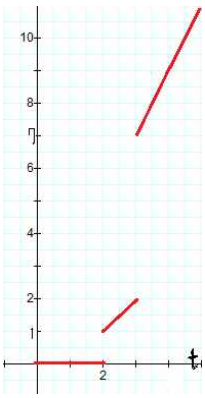
48. $y(t)=(t-t^{2}+\frac{2}{3}t^{3})e^{t}$

49.



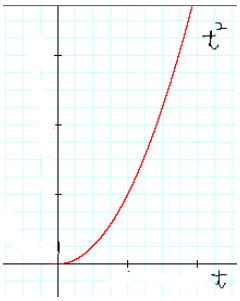
$$\begin{cases}0, & 0\leqq t<1\\ 1, & 1\leqq t<2\\ 2, & 2\leqq t<3\\ 0, & t\geqq 3\end{cases}$$

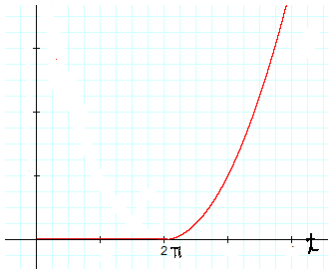
50.



$$\begin{cases}0 & 0\leqq t<2\\ t-1 & 2\leqq t<3\\ 2t+1 & t\geqq 3\end{cases}$$

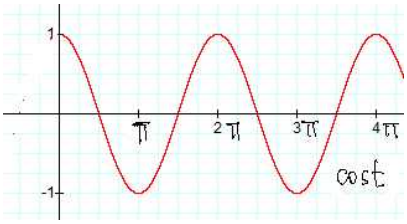
51.

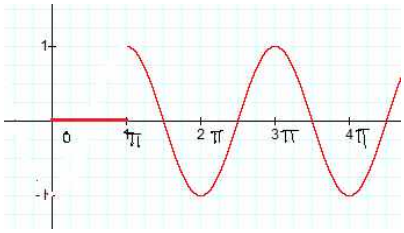




$$\begin{cases} 0 & 0 \leqq t < 2\pi \\ f(t-2\pi) = (t-2\pi)^2 & t \geqq 2\pi \end{cases}$$

52.





$$\begin{cases} 0 & 0 \leqq t \leqq \pi \\ f(t-\pi) = \cos(t-\pi) & \\ \quad = \cos(\pi - t) & t \geqq \pi \\ \quad = -\cos t & \end{cases}$$

53. $\dfrac{2}{s^3}e^{-s}$

54. $(\dfrac{2}{s^3} - \dfrac{2}{s})e^{-2s}$

55. $\dfrac{1}{s^2}e^{-\pi s} - \dfrac{1}{s^2}e^{-3\pi s} - \dfrac{2\pi}{s}e^{-3\pi s}$

56. $\dfrac{1}{s}(e^{-s} + e^{-2s} - 2e^{-3s})$

57. $(\dfrac{1}{s^2} + \dfrac{1}{s})e^{-2s} + (\dfrac{1}{s^2} + \dfrac{5}{s})e^{-3s}$

58. t^2e^{-2t}

59. $\dfrac{1}{\sqrt{3}}\sin\sqrt{3}(t-2)e^{-(t-2)}u(t-2)$

60. $e^{t-1}\cos\sqrt{2}(t-1) \bullet u(t-1)$

61. $\dfrac{1}{2}\sinh 2t$

62. $e^{3(t-2)}\cosh\sqrt{6}\,(t-2)u(t-2)$

63. $e^{(t-1)}u(t-1)+e^{(t-2)}u(t-2)+e^{(t-3)}u(t-3)+e^{(t-4)}u(t-4)$

64. $1-\cos t-(1+\cos t)u(t-\pi)$

65. $(\cos t-\cos\sqrt{2}\,t)+(\cos t+\cos\sqrt{2}\,(t-\pi))u(t-\pi)$

66. $e^{2t}-e^{t}+(\frac{1}{2}-e^{t-3}+\frac{1}{2}e^{2(t-3)})u(t-3)$

67. $\cos t+2t-2\sin t-2[2t+3\sin t-\pi-1+(\pi-1)\cos t]u(t-\pi)$

68. $\frac{1}{17}[\cos t+4\sin t-(\cos t+\frac{9}{2}\sin t)e^{-\frac{1}{2}t}]$

$-\frac{1}{17}[\cos t+4\sin t-(\cos t+\frac{9}{2}\sin t)e^{-\frac{1}{2}(t-\pi)}]u(t-\pi)$

69. $[-1+\frac{1}{2}\cos t(t-1)+\frac{1}{4}e^{-(t-1)}+\frac{1}{4}e^{(t-1)}]u(t-1)$

$+[-1+\frac{1}{2}\cos t(t-2)+\frac{1}{4}e^{-(t-2)}+\frac{1}{4}e^{(t-2)}]u(t-2)$

70. $i(t)=-\frac{1}{10}+t+\frac{1}{10}e^{-10t}-(\frac{19}{10}+(t-2)-\frac{19}{10}e^{-10(t-2)})u(t-2)$

[7.3 합성곱과 주기함수의 라플라스 변환]

71. $\frac{4s}{(s^2+4)^2}$

72. $\frac{-2(5s^2-1)}{(s^2+1)^3}$

73. $\frac{s^2-2s}{(s^2+2s+2)^2}$

74. $\frac{2s+2}{(s^2+2s+2)^2}$

75. $-\frac{1}{2}t\sin t-\frac{1}{2}(\sin t-t\cos t)$

76. $y(t)=2\cos 2t+\sin 2t+\frac{1}{4}t\sin 2t$

77. $\frac{2}{s^3}\frac{1}{s^2+1}$

78. $\frac{1}{s+1}\frac{s}{s^2+4}$

79. $\frac{1}{s^2(s-1)}$

80. $\frac{s}{(s^2+1)(s^2+1)}$

81. $t+e^{-t}-1$

82. $\frac{1}{2}(-\cos t+\sin t+e^{t})$

83. $\frac{1}{2}te^{t}-\frac{1}{2}e^{t}+\frac{1}{2}\cos t$

84. $\frac{1}{2}(t^2-2t-2e^{-t}+2)$

85. $\frac{2}{5}e^{-t}\cos t+\frac{1}{5}e^{-t}\sin t-\frac{2}{5}\cos t+\frac{1}{5}\sin t$

86. $-e^{-2t}+12te^{-2t}+te^{-2t}*g(t)$

87. $-1+t+\frac{2}{3}e^{t}+\frac{4}{3}e^{-\frac{1}{2}t}\cos\frac{\sqrt{3}}{2}t$

88. $2e^{-t}$

89. $1-\cos t+\sin t$

90. $-1+e^{-t}+2t^2e^{-t}$

91. $\frac{1}{3}e^{-t}-\frac{1}{3}e^{\frac{1}{2}t}\cos t\frac{\sqrt{3}}{2}t+\frac{1}{\sqrt{3}}e^{\frac{1}{2}t}\sin\frac{\sqrt{3}}{2}t$

92. $(1-e^{-2(t-3)})u(t-3)$

93. $100(\frac{1}{2}-t-\frac{1}{2}\cos\sqrt{2}t+\frac{1}{\sqrt{2}}\sin\sqrt{2}t)$
$+100\left\{\left(\frac{1}{2}+(t-1)-\frac{1}{2}\cos\sqrt{2}(t-1)-\frac{1}{\sqrt{2}}\sin\sqrt{2}(t-1)\right)u(t-1)\right\}$

94. $\frac{1-se^{-s}-e^{-s}}{s^2(1-e^{-s})}$

95. $\frac{1+e^{-\pi s}}{(1-e^{-\pi s})(1+s^2)}$

96. $\frac{1}{e^{-s}+1}(\frac{e^{-s}}{s}+\frac{e^{-s}-1}{s^2})$

[7.4 라플라스 변환의 성질]

97. $\begin{cases}0 & t<\pi\\ -\frac{1}{3}\sin 3t & t\geqq\pi\end{cases}$

98. $\begin{cases}0 & t<\pi\\ -\sin t & \pi\leqq t<2\pi\\ -2\sin t & t\geqq 2\pi\end{cases}$

99. $\begin{cases}\cosh t & t<1\\ \cosh t+10\sinh(t-1) & t\geqq 1\end{cases}$

100. $\begin{cases}e^t-1 & t<2\\ e^t+e^{t-2}-2 & t\geqq 2\end{cases}$

101. $-\frac{2}{5}\cos t+\frac{1}{5}\sin t+(\frac{2}{5}\cos t+\frac{1}{5}sint)e^{-t}-e^{-(t-\pi)}\sin t\,u(t-\pi)$

102. $\frac{2}{\sqrt{31}}\sin\frac{\sqrt{31}}{4}(t-\frac{\pi}{2})e^{-\frac{1}{4}(t-\frac{\pi}{2})}u(t-\frac{\pi}{2})$

103. $x_1 = c_1\cos 2t + c_2\sin 2t$

$x_2 = -c_1\sin 2t + c_2\cos 2t$

104. $x_1 = (c_1\cos\frac{\sqrt{7}}{2}t + c_2\sin\frac{\sqrt{7}}{2}t)e^{\frac{5}{2}t}$

$x_2 = (-(\frac{3}{4}c_1 + \frac{\sqrt{7}}{4}c_2)\cos\frac{\sqrt{7}}{2}t + (\frac{\sqrt{7}}{4}c_1 - \frac{3}{4}c_2)\sin\frac{\sqrt{7}}{2}t)e^{\frac{5}{2}t}$

105. $x_1 = -\frac{1}{2} - \frac{13}{2}e^{2t} + 7(\cosh\sqrt{2}\,t)e^t + \frac{10}{\sqrt{2}}(\sinh\sqrt{2}\,t)e^t$

$x_2 = -\frac{1}{2} + \frac{7}{2}e^{2t} - 3(\cosh\sqrt{2}\,t)e^t - \frac{4}{\sqrt{2}}(\sinh\sqrt{2}\,t)e^t$

106. $x = -\frac{1}{3}e^{-2t} + \frac{4}{3}e^t,\ y = \frac{1}{3}e^{-2t} + \frac{2}{3}e^t$

107. $y_1 = 1 - \cos t + \sin t + \sin(t-2)u(t-2) + (1 - \cos(t-2))u(t-2)$

$y_2 = (\cos t + \sin t - 1) + (\cos(t-2) + \sin(t-2) - 1)u(t-2)$

108. $y_1 = -2\cos t + \frac{7}{2}sint - \frac{1}{2}t\cos t + e^t + e^{-t}$

$y_2 = e^t + e^{-t} + 2\cos t - \frac{7}{2}\sin t + \frac{1}{2}t\cos t$

109. $y = -\frac{1}{2} + \frac{1}{2}e^t - \frac{1}{2}te^t$

$x = \frac{13}{2} + 5t + \frac{3}{4}t^2 - \frac{13}{2}e^t + \frac{5}{2}te^t$

Chapter 08 연습문제 정답

[8.1 벡터]

1. $\overrightarrow{AB} = < -1, 5, -11 >$, $|\overrightarrow{AB}| = \sqrt{147}$

2. $3\boldsymbol{a} - 2\boldsymbol{b} = < -7, 8, 7 >$, $|3\boldsymbol{a} - 2\boldsymbol{b}| = \sqrt{49 + 64 + 49} = \sqrt{162}$

   $3\boldsymbol{a} - 2\boldsymbol{b}$의 단위벡터 $= \dfrac{3\boldsymbol{a} - 2\boldsymbol{b}}{|3\boldsymbol{a} - 2\boldsymbol{b}|} = < -\dfrac{7}{\sqrt{162}}, \dfrac{8}{\sqrt{162}}, \dfrac{7}{\sqrt{162}} >$

3. $B = (3, -6, 4)$

4. $A = (-1, -2, -1)$

5. ②

6. $t = 1$

7. $\boldsymbol{b} = < \dfrac{2}{\sqrt{14}}, -\dfrac{4}{\sqrt{14}}, -\dfrac{6}{\sqrt{14}} >$

8. $\boldsymbol{a} = -\dfrac{5}{2}\boldsymbol{b} + \dfrac{1}{2}\boldsymbol{c}$

9. $k_1 = -\dfrac{5}{31}$, $k_2 = \dfrac{81}{31}$, $k_3 = \dfrac{27}{31}$

10. $\overrightarrow{PM} = \dfrac{1}{2}\vec{a} + \dfrac{1}{2}\vec{b}$

11. $k = \pm \dfrac{2}{|\vec{a}|}$

12 $k = \pm \sqrt{6}$

13. $k = -1$

[8.2 내적]

14. 6

15. 27

16. -9

17. $c=-9$

18. $\boldsymbol{c}=\ <\ -\frac{1}{8},\ 1,\ -\frac{7}{8}>$

19. $\cos\alpha=\frac{2}{\sqrt{56}},\ \cos\beta=-\frac{6}{\sqrt{56}},\ \cos\gamma=\frac{4}{\sqrt{56}}$

$$\begin{cases}\alpha=\cos^{-1}\left(\frac{2}{\sqrt{56}}\right)\approx 74.49864^\circ\\ \beta=\cos^{-1}\left(-\frac{6}{\sqrt{56}}\right)\approx 143.30077^\circ\\ \gamma=\cos^{-1}\left(\frac{4}{\sqrt{56}}\right)\approx 57.68847^\circ\end{cases}$$

20. $\cos A=\frac{50}{\sqrt{66}\ \sqrt{40}}=\frac{5\sqrt{165}}{66}$

$\cos B=\frac{16}{\sqrt{66}\ \sqrt{6}}=\frac{8\sqrt{11}}{33}$

$\cos C=\frac{-10}{\sqrt{40}\ \sqrt{6}}=-\frac{\sqrt{15}}{6}$

21. $proj_{\boldsymbol{a}}\boldsymbol{b}=\frac{9}{29}<2,3,-4>$, 크기 : $\frac{9}{\sqrt{29}}$

22. $3920\frac{\sqrt{3}}{3}\ (J)$

23. $proj_{\boldsymbol{a}^{\perp}}\boldsymbol{b}=\frac{1}{29}<11,2,7>$

[8.3 외적]

24. $\sqrt{22}$

25. $<30,-12,-6>$

26. $<-2k,-7k,5k>$, $(k\neq 0)$

27. 16

28. 3

29. $c \neq \frac{5}{2}$

30. $\sqrt{281}$

31. $\frac{3}{2}\sqrt{13}$

[8.4 3차원 공간에서의 직선과 평면]

32. $< 1, 2, -2 > + t < 4, -1, -2 >$

33. $< x, y, z > = < 1, -2, 3 > + t < 5, 3, -2 >$

34. $\frac{x-3}{1} = \frac{y-2}{3} = \frac{z+1}{5}$

35. $< x, y, z > = < 2, 1, -1 > = t < 1, 2, 3 >$ 또는 $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z+1}{3}$

36. $(\frac{13}{3}, 2, -\frac{5}{3})$

37. $12 < 1, 0, 0 >$ 또는 $12 < -1, 0, 0 >$

38. $x + z = 5$

39. $4x - y + z - 5 = 0$

40. $2x + 3y - 4z - 13 = 0$

41. $39x + 22y + 36z - 112 = 0$

42. $x = 1$

43. $\begin{cases} x = \frac{13}{5} + \frac{7}{5}t \\ y = \frac{6}{5} + \frac{9}{5}t \\ z = t \end{cases}$

44. $(-\frac{2}{5}, -\frac{4}{5}, -\frac{21}{5})$

[8.5 벡터공간]

45. 생략

46. V 는 벡터공간이 아니다.

47. 생략

48. $\mathbb{R}^3$는 주어진 합과 스칼라 곱의 연산에 대해서는 벡터공간을 이루지 못한다.

49. V는 $\mathbb{R}^4$의 부분공간이다.

50. V는 $\mathbb{R}^4$의 부분공간이다.

51. A는 P_3의 부분공간이다.

52. B는 $\mathbb{R}^2$의 부분공간이 아니다.

53. A는 $\mathbb{R}^2$의 부분공간이 아니다.

54. B는 $\mathbb{R}^2$의 부분공간이다.

55. 일차독립

56. 일차독립

57. 일차독립

58. 일차종속

59. $< x, y, z > = (x-y) < 1,0,0 > + (y-z) < 1,1,0 > + z < 1,1,1 > \mathbb{R}^3$

60. $< 1,2 > = \frac{3}{2} < 1,1 > + (-\frac{1}{2}) < 1,-1 >$

61. 49번-3차원, 50번-2차원, 51번-2차원

Chapter 09 연습문제 정답

[9.1 행렬과 연산]

1. $x = \pm 3,\ y = \pm 12$ (복호동순)

2. $-\frac{1}{3}\mathrm{A} = \begin{pmatrix} -\frac{1}{3} & -\frac{2}{3} \\ -1 & -\frac{4}{3} \end{pmatrix}$

   $\mathrm{A} + 2\mathrm{B} = \begin{pmatrix} 11 & 14 \\ 17 & 20 \end{pmatrix}$

   $\mathrm{A} - \mathrm{B} = \begin{pmatrix} -4 & -4 \\ -4 & -4 \end{pmatrix}$

   $\mathrm{A} - \mathrm{B}^{T} = \begin{pmatrix} -4 & -5 \\ -3 & -4 \end{pmatrix}$

3. $\mathrm{AB} = \begin{pmatrix} 14 & -10 \\ 28 & -14 \end{pmatrix},\ \mathrm{BA} = \begin{pmatrix} 18 & 24 \\ -17 & -18 \end{pmatrix}$

   $\therefore\ \mathrm{AB} \neq \mathrm{BA}$

4. $\mathrm{B}^{T}\mathrm{A}^{T} = \begin{pmatrix} 5 & 2 \\ 10 & -5 \end{pmatrix}\begin{pmatrix} 3 & 7 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 23 & 39 \\ 10 & 60 \end{pmatrix}$

   $(AB)^{T} = B^{T}A^{T}$

5. 생략

6. $\mathrm{A} = \begin{pmatrix} a & b \\ 2a & 2b \end{pmatrix},\ \mathrm{B} = \begin{pmatrix} -b & b \\ a & -a \end{pmatrix}$

7. 생략

[9.2 연립 선형방정식과 가우스 소거법]

8. $x_1 = \frac{27}{8},\ x_2 = \frac{9}{8},\ x_3 = \frac{15}{8}$

9. $x_1 = -\frac{3}{5}t + 1$

   $x_2 = \frac{1}{5}t$

   $x_3 = t$

10. $x_1 = 1, x_2 = t, x_3 = 4 - 2t, x_4 = t$

11. $x_1 = x_2 = x_3 = 0$

12. $x_1 = t,\ x_2 = -2t, x_3 = t$

[9.3 행렬의 계수]

13. $rank\ \mathrm{A} = 3$

14. $rank\ \mathrm{B} = 3$

15. $rank\ \mathrm{C} = 2$

16. 일차종속

17. 일차독립

18. 자명한 해를 갖는다.

19. $a = 2$

20. $a - b - c = 0$

[9.4 행렬식]

21. $x - 2y + z$

22. 0

23. 0

24. 34

25. $x = 1$

26. $x = 2,\ -2 \pm \sqrt{-3}$

27. k

28. $-k$

29. $6k$

30. $-k$

31. 24

32. 24

33. -6

34. 0

[9.5 역행렬]

35. $A^{-1}=\begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$

36. $\mathrm{A}^{-1}=\begin{pmatrix} 1 & -\frac{5}{2} & \frac{3}{2} \\ 2 & -8 & 5 \\ -1 & 5 & -3 \end{pmatrix}$

37. $\mathrm{A}^{-1}=-\frac{1}{3}\begin{pmatrix} -12 & -3 & 3 \\ 31 & 7 & -9 \\ -20 & -5 & 6 \end{pmatrix}$

38. $\mathrm{A}^{-1}=\begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$

39. $\mathrm{A}^{-1}=\begin{pmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{pmatrix}$

40. $x_1=\frac{61}{53}, x_2=-\frac{51}{53}, x_3=\frac{13}{53}$

41. $\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 7 \\ 25 \\ -15 \end{pmatrix}$

42. $\lambda = 4,\ -1$

43. $\mathrm{A} = \frac{1}{26}\begin{pmatrix} 8 & -1 \\ 3 & 11 \end{pmatrix}$

Chapter 10 연습문제 정답

[10.1 크래머의 법칙]

1. $x_1 = \frac{61}{53},\ x_2 = -\frac{51}{53},\ x_3 = \frac{13}{53}$

2. $x_1 = 7,\ x_2 = 25,\ x_3 = -15$

3. $k \neq \frac{6}{5}$인 경우 $x_1 = \frac{3k-2}{-5k+6},\ x_2 = \frac{3k-6}{-5k+6}$ (유일해)

   $k = \frac{6}{5}$인 경우 해를 갖지 않는다.

[10.2 고윳값과 고유벡터]

4. $\lambda = 1 \pm \sqrt{6}$

   $\mathrm{K}_1 = \begin{pmatrix} 2 \\ \sqrt{6} \end{pmatrix},\ K_2 = \begin{pmatrix} -2 \\ \sqrt{6} \end{pmatrix}$

5. $\lambda = 4,\ -4$

   $\mathrm{K}_1 = \begin{pmatrix} 8 \\ 3 \end{pmatrix},\ \mathrm{K}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

6. $\lambda = 0$(중근)

   $\mathrm{K} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

7. $\lambda_1 = 1,\ \lambda_2 = -1,\ \lambda_3 = 2$

   $\mathrm{K}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},\ \mathrm{K}_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix},\ \mathrm{K}_3 = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$

8. $\lambda_1 = 1,\ \lambda_2 = 2,\ \lambda_3 = 3$

   $\mathrm{K}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},\ \mathrm{K}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix},\ \mathrm{K}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

9. $\lambda_1 = 1,\ \lambda_2 = 2,\ \lambda_3 = 3$

   $\mathrm{K}_1 = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix},\ \mathrm{K}_2 = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix},\ \mathrm{K}_3 = \begin{pmatrix} -5 \\ -7 \\ 3 \end{pmatrix}$

[10.3 직교행렬]

10. $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$

11. $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

12. $\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$

13. $\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}$

[10.4 대각화]

14. $\begin{pmatrix} 1 & -1+3^5 \\ 0 & 3^5 \end{pmatrix}$

15. $\begin{pmatrix} 5^6 & 2\times 5^6 \\ 2\times 5^6 & 4\times 5^6 \end{pmatrix}$

16. $\begin{pmatrix} \frac{5}{2}-2^{11}+\frac{5}{2}\times 3^9 & -2+2^{10} & -\frac{1}{2}-2^{10}+\frac{5}{2}\times 3^9 \\ \frac{5}{2}-3\times 2^{10}+\frac{7}{2}\times 3^9 & -2+3\times 2^9 & -\frac{1}{2}-3\times 2^9+\frac{7}{2}\times 3^9 \\ -\frac{5}{2}+2^{11}-\frac{1}{2}\times 3^{10} & 2-2^{10} & \frac{1}{2}+2^{10}-\frac{1}{2}\times 3^{10} \end{pmatrix}$

17. $\begin{pmatrix} 1 & 2 & -3+3\cdot 2^{11} \\ 0 & -1 & 0 \\ 0 & 0 & 2^{11} \end{pmatrix}$

18. E^{13}은 대각화를 이용한 방법으로 계산할 수 없다.

19. $\begin{pmatrix} \frac{1}{3}5^{15}-\frac{2}{3} & -\frac{1}{3}5^{15}-\frac{1}{3} & \frac{1}{3}5^{15}+\frac{1}{3} \\ -\frac{1}{3}5^{15}-\frac{1}{3} & \frac{1}{3}5^{15}-\frac{2}{3} & -\frac{1}{3}5^{15}-\frac{1}{3} \\ \frac{1}{3}5^{15}+\frac{1}{3} & -\frac{1}{3}5^{15}-\frac{1}{3} & \frac{1}{3}5^{15}-\frac{2}{3} \end{pmatrix}$

20. $\begin{pmatrix} 4\times3^{15}+16\times6^{15}+9^{16} & 4\times3^{15}-8\times6^{15}-2\times9^{16} & 2\times3^{15}-16\times6^{15}+2\times9^{16} \\ 4\times3^{15}-8\times6^{15}-2\times9^{16} & 4\times3^{15}+4\times6^{15}+4\times9^{16} & 2\times3^{15}+8\times6^{15}-4\times9^{16} \\ 2\times3^{15}-16\times6^{15}+2\times9^{16} & 2\times3^{15}+8\times6^{15}-4\times9^{16} & 3^{15}+16\times6^{16}+4\times9^{16} \end{pmatrix}$

21. $\frac{3}{10}x'^2-\frac{1}{5}y'^2=1$

22. $-x'^2+y'^2=5$

23. $\frac{x'^2}{\left(\sqrt{\frac{31}{3}}\right)^2}+\frac{\left(y'+\frac{1}{3}\right)^2}{\left(\frac{\sqrt{31}}{3}\right)^2}=1$