

MSE, 미적분학

[연습문제 답안 이용 안내]

- 본 연습문제 답안의 저작권은 한빛아카데미(주)에 있습니다.
- 이 자료를 무단으로 전제하거나 배포할 경우 저작권법 136조에 의거하여 최고 5년 이하의 징역 또는 5천만원 이하의 벌금에 처할 수 있고 이를 병과(併科)할 수도 있습니다.

Chapter 09 연습문제 답안

《Section 9.1》

1.
 - (a) $\sqrt{1+4+81} = \sqrt{86}$
 - (b) $\sqrt{4+9+49} = \sqrt{62}$
 - (c) 7
 - (d) 2
 - (e) $\sqrt{4+9} = \sqrt{13}$
 - (f) $\sqrt{4+49} = \sqrt{53}$
 - (g) $\sqrt{y^2+x^2}$
 - (h) $\sqrt{y^2+x^2}$
2. $\overrightarrow{AF} = (-4, 0, 3), \overrightarrow{HB} = (4, 5, 0), \overrightarrow{HE} = (4, 5, 3)$
3. $(0, 2)$
4. $(-3, 2), (3, -2), (6, -4), (-6, 4)$
5. $B - A = (-3, -3)$
6. $(3, 1, 6) + (1, 0, 4) = (4, 1, 10)$
7. $(3\cos 120^\circ, 3\sin 120^\circ) = (-\frac{3}{2}, \frac{3}{2}\sqrt{3})$

《Section 9.2》

1. (a) \overrightarrow{DB}
 (b) \overrightarrow{DB}
 (c) $\overrightarrow{AB} + \overrightarrow{CB} = \overrightarrow{AB} + \overrightarrow{DA} = \overrightarrow{DB}$
 (d) $\overrightarrow{AB} - \overrightarrow{CB} = \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$
 (e) $\vec{0}$
2. $(6, 7, 8)$
3. $\overrightarrow{AB} = (1, 4, -4), \overrightarrow{CD} = (-2, -8, 8)$
 $\overrightarrow{CD} = -2\overrightarrow{AB}$ 이므로 평행
4. $\overrightarrow{AB} = (3, 6, -4), \overrightarrow{QP} = (10, y, z - 2)$.
 $10/3 = y/6, y = 20$
 $10/3 = (z - 2)/(-4), z = -34/3$
5. 아니요
6. $A = \frac{1}{10}(Q + 9P), B = \frac{1}{10}(2Q + 8P), C = \frac{1}{10}(3Q + 7P)$
7. $a = -1, b = 2$
8. $\overrightarrow{AE} + \overrightarrow{BF} + \overrightarrow{CD} = E - A + F - B + D - C$
 $= \frac{1}{2}(B + C) - A + \frac{1}{2}(A + C) - B + \frac{1}{2}(A + B) - C = \vec{0}$
9. (a) $\sqrt{9 + 1 + 25} = \sqrt{35}$
 (b) $\sqrt{\pi^2 + \pi^2 + \pi^2 + \pi^2 + \pi^2} = \pi\sqrt{5}$
10. $(2/\sqrt{104}, -6/\sqrt{104}, 8/\sqrt{104})$
11. $-5\vec{u}/|\vec{u}|$
12. $\overrightarrow{BC} = 12\overrightarrow{BA}_{normalized} = 12(0, -1/\sqrt{17}, -4/\sqrt{17})$
 $C - B = (0, \frac{-12}{\sqrt{17}}, \frac{-48}{\sqrt{17}}), C = (1, 2 - \frac{12}{\sqrt{17}}, 6 - \frac{48}{\sqrt{17}})$

13. $\|\vec{u}\| = \sqrt{38}, \|217\vec{u}\| = 217\sqrt{38}$

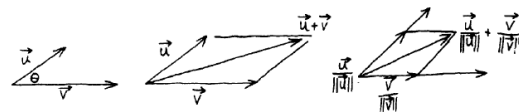
14. $(\cos\theta, \sin\theta)$

15. $\vec{u} = 3\vec{AB}_{normalized} = 3(-4, 0, -6)_{normalized}$
 $= 3(-4/\sqrt{52}, 0, -6/\sqrt{52}) = (-12/\sqrt{52}, 0, -18/\sqrt{52})$

16. $\vec{AB} = (-5, -6, -7), \|\vec{u}\| = 1/(25 + 36 + 49) = 1/110,$
 $\vec{u} = \frac{1}{110}\vec{AB}_{normalized} = (\frac{-5}{110\sqrt{110}}, \frac{-6}{110\sqrt{110}}, \frac{-7}{110\sqrt{110}})$

17. $\vec{u} - 2\vec{v} = 5\vec{j} - 3\vec{k}$
 $\|\vec{u}\| = \sqrt{4 + 9 + 1} = \sqrt{14}$
 $\vec{u}_{normalized} = (2/\sqrt{14})\vec{i} + (3/\sqrt{14})\vec{j} - (1/\sqrt{14})\vec{k}$

18. r^4

19. 

《Section 9.3》

1. $\vec{u} \cdot \vec{v} = 5 + 12 - 15 = 2$
 $\vec{u} \cdot \vec{v}$ 의 크기로 예 각

2. $\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos 180 = 30(-1) = -30$

3. $\cos A = \frac{\vec{AB} \cdot \vec{AC}}{\|\vec{AB}\| \|\vec{AC}\|}, \vec{AB} = (1, -3, 9),$
 $\vec{AC} = (3, -1, 5),$
 $\cos A = \frac{51}{\sqrt{91} \sqrt{35}} = 0.904$
 $\cos^{-1} 0.904 = 25^\circ$

4. (a) $\cos \theta_1 = \frac{u_1}{|\vec{u}|}, \cos \theta_2 = \frac{u_2}{|\vec{u}|}, \cos \theta_3 = \frac{u_3}{|\vec{u}|}$
 (b) $(\cos \theta_1, \cos \theta_2, \cos \theta_3) = \left(\frac{u_1}{|\vec{u}|}, \frac{u_2}{|\vec{u}|}, \frac{u_3}{|\vec{u}|} \right) = \frac{\vec{u}}{|\vec{u}|}$

5. (a) AB 의 기울기 $= \frac{5}{3}, CD$ 의 기울기 $= -\frac{3}{5}$
 (b) $\vec{AB} \cdot \vec{CD} = 0$

6. $\vec{AB} = (6, 5), \vec{u} = (-5, 6), \vec{BC} = 7\vec{u}/\|\vec{u}\|,$
 $C - B = (-35/\sqrt{61}, 42/\sqrt{61}), C = (8 - \frac{35}{\sqrt{61}}, 9 + \frac{42}{\sqrt{61}})$

7. $\cos \theta = \frac{26}{\sqrt{53} \sqrt{17}}$

8. $\vec{u} \cdot (a\vec{v} - b\vec{u}) = a(\vec{u} \cdot \vec{v}) - b(\vec{u} \cdot \vec{u}) = (\vec{u} \cdot \vec{u})(\vec{u} \cdot \vec{v}) - (\vec{v} \cdot \vec{u})(\vec{u} \cdot \vec{u}) = 0$
 $\vec{u} \perp a\vec{u} - b\vec{v}$

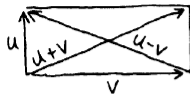
9. $\| -6\vec{u} \| = 6\|\vec{u}\| = 18,$
 $\vec{u} \cdot 3\vec{u} = 3(\vec{u} \cdot \vec{u}) = 3\|\vec{u}\|^2 = 27$
 $\|\vec{u} - \vec{v}\| = \sqrt{(\vec{u} - \vec{v}) \cdot (\vec{u} - \vec{v})} = \sqrt{\|\vec{u}\|^2 - 2\vec{u} \cdot \vec{v} + \|\vec{v}\|^2} = \sqrt{9 - 10 + 4} = \sqrt{3}$

10. (a) 33

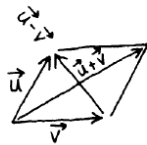
- (b) 의미 없음.
 (c) $(5\sqrt{42}, 2\sqrt{42}, 3\sqrt{42} - 4\sqrt{42})$
 (d) 의미 없음.
 (e) $2/\sqrt{54}$
 (f) $(132, -99, 33, -132)$
 (g) 의미 없음.

11.

(a)



(b)



12.

- (a) $-7/\sqrt{11}$
 (b) $-7/\sqrt{29}$

13.

$$96/\sqrt{153}$$

14.

- (a) 6
 (b) -6
 (c) 24

15.

$$-3$$

16.

인정될 수 없다.

17.

아니오.

18.

$$\vec{u}$$

19.

$$\frac{20}{29}\vec{i} - \frac{8}{29}\vec{j}$$

20.

$$\vec{p}$$

《Section 9.4》

1. $\vec{u} \times \vec{v}$: 이 페이지 위
 $\vec{p} \times \vec{q}$: 동쪽
 $s \times t = 0$

2. $\vec{u} = \vec{0}, \vec{v} = \vec{0}$

3. $\vec{0}$

4. $\vec{a} \times \vec{x} = \vec{b}$ 이면, $\vec{b} \perp \vec{a}$ 이고 $\vec{b} \perp \vec{x}$ 이다.
 이때 $\vec{a} \cdot \vec{b} \neq 0$ 이면 \vec{a} 와 \vec{b} 는 수직일 수 없다.
 따라서 이를 만족하는 \vec{x} 는 없다.

5. (a) 내적은 벡터끼리의 연산이므로.

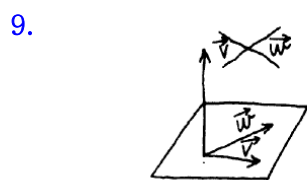
(b) 0

(c) 0

6. $\vec{0}$

7. $\vec{0}$

8. $15(\vec{u} \times \vec{v})$



10. (a) $(-11, -12, 27)$

(b) $-17\vec{i} + 13\vec{j} + \vec{k}$

(c) $(0, 0, 21)$

(d) $(4, 2, 11)$

11. $(13, -11, 3)$

12. $\cos\theta = \frac{5}{\sqrt{14}\sqrt{30}}, \sin\theta = \frac{\sqrt{395}}{\sqrt{14}\sqrt{30}}$

13. (a) $\vec{i} + 2\vec{j}, 6\vec{i} + 3\vec{j} - 3\vec{k}$
(b) $(-3, 27, 11)$

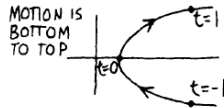
14. $\frac{1}{2} \sqrt{(24)^2 + (31)^2 + (30)^2}$

《Section 9.5》

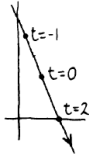
1. $(1, 2, 3) \cdot (1, 4, -3) = 0$
2. 16
3. $\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD} = 12 \neq 0$ (한 평면에 있지 않음)
4. 음수
5. 0
6.
 - (a) -5
 - (b) 5
 - (c) 5
 - (d) -300
 - (e) 0
 - (f) 0

《Section 9.6》

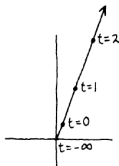
1. (a) $x = y^2 + 5$



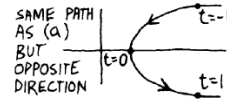
(c) $y = 8 - 2x$



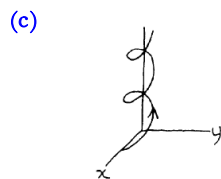
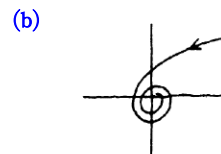
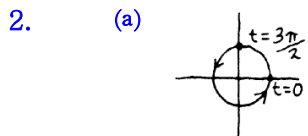
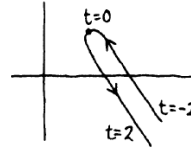
(e) $y = 2x$



(b) $x = y^2 + 5$



(d) $x = y^2 + 5$



3. (a) $y = -3\sin t$

(b) $y = 7 + 3\sin t$

(c) $y = 3\sin 2\pi t$

4. (a) 지나지 않음

(b) $t = -1$

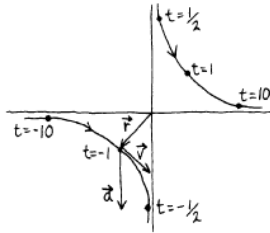
5. \vec{r}_1 의 자취를 5초 후에 \vec{r}_2 가 따라감

6. 7

7. $3t^2\vec{i} + 2\vec{j} - \sin t\vec{k}$
8. $\vec{v} = -i - \pi\vec{j}$
9. (a) 6
(b) $\vec{v} = -6\vec{i}$
10. $|\vec{v}| = \sqrt{1+4t^2}$
11. $|\vec{t}| = \sqrt{4t^2([\sin t]^2 + \cos t^2)^2} = 2|t|$
12. (a) \vec{v} 는 일정하므로 직선
(b) $|\vec{v}| = \sqrt{29}$
(c) $(-1 + \frac{6}{\sqrt{29}}t)\vec{i} + (1 - \frac{4}{\sqrt{29}}t)\vec{j} + \frac{8}{\sqrt{29}}t\vec{k}$
13. (a) 원점에서 움직이지 않음
(b) 움직이지 않음
14. (a) $(t^2 - 8)\vec{i} + (\frac{5}{3}t^3 - 41)\vec{j} + (6t - 12)\vec{k}$
(b) $(4/\sqrt{452})\vec{i} + (20/\sqrt{452})\vec{j} + (6/\sqrt{452})\vec{k}$
15. (a) $x^2/9 + y^2/4 = 1$
(b) 속력 = $\sqrt{5\sin^2 t + 4}$, 최대 속력 = 3

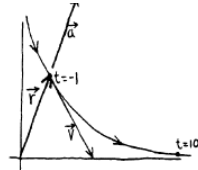
《Section 9.7》

1.



$t = -1$ 일 때, 가속한다.
 변화량 : $\sqrt{2}$
 힘 : $2m$ (질량이 m 일 때)

2.



속력은 줄어든다.

3.

$$\begin{aligned}\vec{r}'' &= \vec{a} = \vec{f}/m \\ \vec{r} \times \vec{r}'' &= \vec{r} \times (\vec{f}/m) = \left(\frac{1}{m}\right)(\vec{r} \times \vec{f}) \\ \vec{f} &\parallel \vec{r}\end{aligned}$$

4.

$$\vec{r} = (4t+1)\vec{i} + \left(-\frac{1}{2}gt^2 + 2t + 2\right)\vec{j}$$

5.

$$\begin{aligned}\text{(a)} \quad \frac{ds}{dt} &= \text{거리의 변화(속력)} = |\vec{v}| \\ \frac{d^2s}{dt^2} &= \text{속력의 변화(가속도)} = a_{\text{tan}} \\ \text{(b)} \quad \frac{d\vec{r}}{ds} &= \frac{d\vec{r}/dt}{ds/dt} = \frac{\vec{v}}{|\vec{v}|}\end{aligned}$$

6.

$$\begin{aligned}\text{(a)} \quad &5 \\ \text{(b)} \quad \vec{a} &= (-5\cos t, -5\sin t), a_{\text{tan}} = 0 \\ \text{(c)} \quad a_{\text{rad}} &= 5\end{aligned}$$

7.

$$\begin{aligned}\text{(a)} \quad &\vec{a} \parallel \vec{v} \\ \text{(b)} \quad &\vec{a} \perp \vec{v}\end{aligned}$$

8. $\begin{pmatrix} 3, 5/4 \end{pmatrix}$
 $\vec{v} = -3\vec{i} + 2\vec{j}, |\vec{v}| = \sqrt{13}$
 $\vec{a} = -6\vec{i} + 3\vec{j}$
 $m|\vec{a}| = m\sqrt{45}$

《복습문제》

1. (a) 5
 (b) $\sqrt{14}$
 (c) $(-11, 0, 22)$
 (d) $\frac{5}{\sqrt{14}\sqrt{45}}$
 (e) $5/\sqrt{45}$
 (f) $(-4/9, 5/9, -2/9)$
 (g) $(-4/\sqrt{45})\vec{i} + (5/\sqrt{45})\vec{j} - (2/\sqrt{45})\vec{k}$
 (h) $(12/\sqrt{14})\vec{i} + (18/\sqrt{14})\vec{j} - (6/\sqrt{14})\vec{k}$


2. 아니요

3. (a) $\vec{j} - 5\vec{k}$ (b) 6

4. (a) 참 (b) 거짓

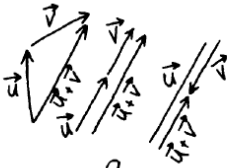
5. -2

6. $\vec{u} \times \vec{v} = (0, 0, | \begin{smallmatrix} u_1 & u_2 \\ v_1 & v_2 \end{smallmatrix} |)$

7. (a) 

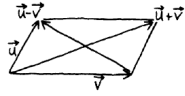
(b) $(\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v}) = |\vec{u}|^2 + |\vec{v}|^2$

8. $|\vec{u} \times \vec{v}| = |\vec{u}| \times |\vec{v}|$

9. 

10. (a) $|\vec{u} + \vec{v}|^2 + |\vec{u} - \vec{v}|^2 = 2\vec{u} \cdot \vec{u} + \vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{u} + 2\vec{v} \cdot \vec{v} = 2|\vec{u}|^2 + 2|\vec{v}|^2$

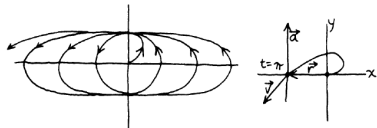
(b)



11. 역풍, $8/\sqrt{42} \text{ mph}$ 감소

12. $\vec{u} \perp \vec{v}$

13.

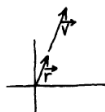


$$\begin{aligned}\vec{v} &= (-t \sin t + \cos t)\vec{i} + \cos t\vec{j} \\ \vec{a} &= (-t \cos t - 2 \sin t)\vec{i} - \sin t\vec{j} \\ (t = \pi)\vec{r} &= -\pi\vec{i}, \vec{v} = -\vec{i} - \vec{j}, \vec{a} = \pi\vec{i} \\ \pi/\sqrt{2} [m/s] \text{ 감소}\end{aligned}$$

14. (a)



(b)



(c) 직선 운동

(d) 등속도 운동