MSE, 미적분학

[연습문제 답안 이용 안내]

Chapter 10 연습문제 답안

《Section 10.1》

1. $(x-4)^2+(y+3)^2+(z-5)^2=50$

2. $x^2+y^2+(z+\frac{1}{2})=\frac{1}{4}$, 중심 $(0,0,-\frac{1}{2})$, 반지름 $\frac{1}{2}$

3. 원점은 외부에 있다.

4. $(x-3)^2+(y-5)^2+(z-6)^2=36$

《Section 10.2》

1. (a) $5(x-5)+3(y-5)+z-4=0$
$5x+3y+z=44$

(b) $a=2, b=5, c=7$
$x/2+y/5+z/7=1$

(c) $x=3$

(d) $z=5$

(e) $2(x-3)+9(y-\pi)-6(z-7)=0$
$2x+9y-6z=9\pi-36$

2. 아니오

$\overrightarrow{AB}\times\overrightarrow{AC}=(1,-4,2)\times(3,1,5)=(-22,1,13)$
$-22(x-1)+(y-3)+13(x+2)=0$
$-22x+y+13z=-45$이지만, 임의의 점 $D(1,2,3)$을 넣으면
$-22(1)+2+13(3)\neq-45$로 만족하지 않는다.

3. $\dfrac{|3(2)-4(3)+2(-4)-6|}{\sqrt{9+16+4}}=\dfrac{20}{\sqrt{29}}$

4. $\dfrac{|3(0)-0+4|}{\sqrt{9+1}}=\dfrac{4}{\sqrt{10}}$

5. $\dfrac{|2(1)+3-(-1)-4|}{\sqrt{4+1+1}}=\dfrac{2}{\sqrt{6}}$
$(x-1)^2+(y-3)^2+(z+1)^2=\dfrac{4}{6}$

6. $\dfrac{|2(0)-0+3(2)-8|}{\sqrt{4+1+9}}=\dfrac{2}{\sqrt{14}}$

7. $D=\dfrac{|0+0+0-1|}{\sqrt{(\frac{1}{a})^2+(\frac{1}{b})^2+(\frac{1}{c})^2}}$, $\dfrac{1}{D^2}=\dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{c^2}$

8. $\overrightarrow{AB}\times\overrightarrow{CD}=(1,1,-1)\times(3,13,3)=(16,-6,10)$
$16(x+1)-6(y-2)+10(z-4)=0$
$16x-6y+10z=12$

《Section 10.3》

1. (a) $x = 1 + t, y = 2 - 2t, z = 3 + 5t$

(b) $x = 1 + 3t, y = 4 - 4t, z = 5 + 6t$

(c) $x = 2 + t, y = 3, z = 4$

(d) $x = 2, y = t, z = 4$

(e) $x = 7t, y = -t, z = 16t$

(f) $x = 1 + t, y = 5 + t, z = 7 - t$

2. $\overrightarrow{AB} = (14, -5)$
$x = -1 + 14t, y = 3 - 5t$

3. (a) $y = 0$

(b) $x = 0, y = 0, z = t$

4. $P = (6, 2, 7), \vec{u} = (4, -1, 8)$이므로
$\vec{u}, \overrightarrow{AP} = (3, 3, 5)$는 모두 $\vec{u} \times \overrightarrow{AP} = (-29, 4, 15)$에 평행하다.
$-29(x-3) + 4(y+1) + 15(z-2) = 0$
$-29x + 4y + 15z = -61$

5. (a) $\vec{u} = (1, -4, 5), \vec{v} = (2, -1, -6)$

(b) $\vec{u} \times \vec{v} = (29, 16, 7)$
$29(x-2) + 16(y-3) + 7(z-6) = 0$
$29x + 16y + 7z = 148$

(c) $x = 4 + 29t, y = -5 + 16t, z = 16 + 7t$

6. $(0, 11, -1)$

7. $x - 2 + 5(y-5) - (z-4) = 0$
$x + 5y - z = 23$

8. $x = 1 + 3t, y = 3 + 2t, z = -2 + 2t$, 한 평면 위에 있지 않음.

9. 직선은 평면위에 있다.

10. (a) 평행

(b) 꼬여있음

(c) $(5, 0, -1)$에서 교차

(d) 일치

11. (a) $2(1+2t)+6(3-t)+2+2t=8$,
$0=-14$
답이 없음.

(b) $2(1+xt)+6(3-t)+2+2t=22, 0=0$
일치

12. $\vec{u}=(2,1,3), \vec{v}=(1,-1,1)$
$\vec{u}\times\vec{v}=(4,1,-3)$
$x=3+4t, y=-1+t, z=-3t$

13. $\vec{u}=(1,2,8)\times(1,-1,2)=(12,6,-3), \vec{v}=(3,-2,8), \vec{u}\cdot\vec{v}=0$(평행)
직선에 포함된 점 $(12,4,0)$대입 시 $(36-8+0\neq 5)$

14. $\vec{u}=(2,-3,1)$
$x=-1+2t, y=1-3t, z=1+t$
$2(-1+2t)-3(1-3)+1+t+1=0, t=\frac{3}{14}$
$P=(-\frac{8}{14},\frac{5}{14},\frac{17}{14})$

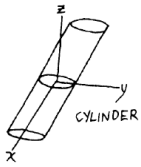
15. $\vec{u}=(1,-1,4)$
$x-4-(y+1)+4(z-4)=0,$
$x-y+4z=21$
$1+t-(2-t)+4(3+4t)=21, t=\frac{5}{9}$
$P=(\frac{14}{9},\frac{13}{9},\frac{47}{9}),$
$\overline{PQ}=\sqrt{(4-\frac{14}{9})^2+(-1-\frac{13}{9})^2+(4-\frac{47}{9})^2}$

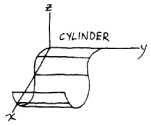
16. $3x-5y-z=28, \frac{12}{\sqrt{35}}$

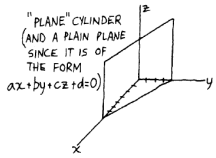
17. (a) $\overrightarrow{AB}=(-1,4,2)//\overrightarrow{CD}=(3,-12,6)$
$\overrightarrow{AB}$와 $\overrightarrow{AC}=(1,5,4)$는 평행하지 않음

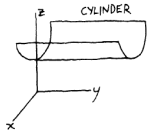
(b) $\overline{AP}=\sqrt{(3-\frac{95}{21})^2+(\frac{61}{21})^2+(2-\frac{148}{21})^2}$

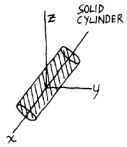
《Section 10.4》

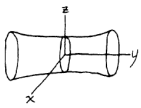
1. 

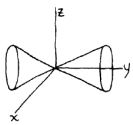
2. 

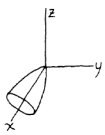
3. 

4. 

5. 

6. 

7. 

8. 

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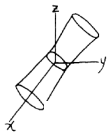

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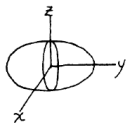

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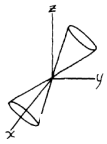

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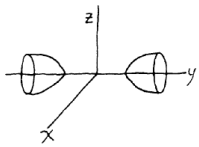

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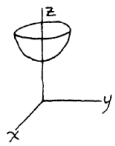

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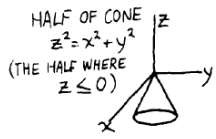

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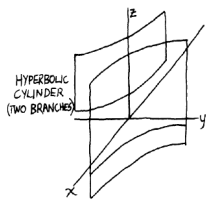


16. 쌍곡포물면, 안장형구조(그리기엔 어려움)

17. $a = b = \frac{1}{2}$

《Section 10.5》

1. (a) $(2, 90, 3)$

(b) $(5, 90, 90)$

2. (a) $(\sqrt{13}, 34, 5)$

(b) $(\sqrt{13}, 214, 5)$

3. (a) $x = 2\cos 150^\circ = 2(-\frac{1}{2}\sqrt{3}) = -\sqrt{3}$

$y = 2\sin 150^\circ = 2(\frac{1}{2}) = 1$

$z = 7$

$(-\sqrt{3}, 1, 7)$

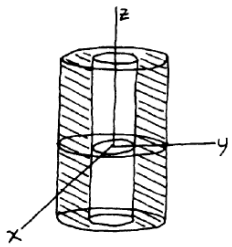
(b) $x = 2\sin 30^\circ \cos 120^\circ = 2(\frac{1}{2})(-\frac{1}{2}) = -\frac{1}{2}$

$y = 2\sin 30^\circ \sin 120^\circ = \frac{1}{2}\sqrt{3}$

$z = 2\cos 30^\circ = \sqrt{3}$

$(-\frac{1}{2}, \frac{\sqrt{3}}{2}, \sqrt{3})$

4. (a)



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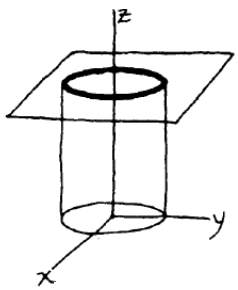
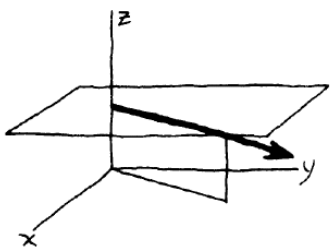
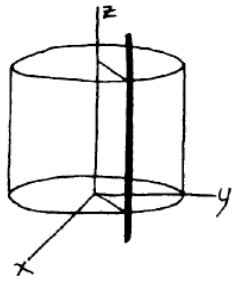


(c)



(d)



5. (a) $r^2 = 4z^2, \rho^2\sin^2\phi(\cos^2\theta + \sin^2\theta) = 4\rho^2\cos^2\phi$, 원뿔모양

(b) $r^2 + z^2 = 10, \rho = \sqrt{10}$, 원통형

6. (a) $z = 0, r = 0$

(b) $\phi = 90, \phi = 0$ 또는 180

7. (a) $\sqrt{x^2+y^2+z^2}$ (직교), $\sqrt{r^2+x^2}$ (원기둥), ρ(구면)

(b) $\sqrt{x^2+y^2}$ (직교), r(원기둥), $\rho\sin\theta$(구면)

《복습문제》

1. (a) $x = 2 - t, y = 3 + t, z = -3 + 3t$

   (b) $\vec{u} \times \vec{v} = (4, 7, -1)$
   $x = 2 + 4t, y = 3 + 7t, z = -3 - t$

   (c) $-(x-2) + y - 3 + 3(z+3) = 0$
   $-x + y + 3z = -8$

   (d) $4(x-2) + 7(y-3) - (z+3) = 0$
   $4x + 7y - z = 32$

2. $t + 5(3 + 2t) - (2 - t) = 1, t = -1$
   $x = -1, y = 1, z = 3$

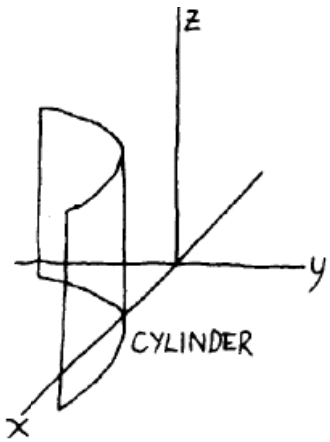
3. $\overrightarrow{AB} = (-3, -4, -4)$,
   $x = 9 - 3t, y = 8 - 4t, z = 7 - 4t$

4. $$\frac{|5(0) + 2(0) - 6(0) - 8|}{\sqrt{25 + 4 + 36}} = \frac{8}{\sqrt{65}}$$

5.

6.

7.



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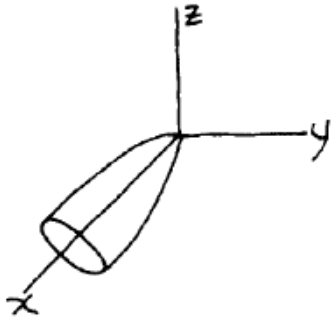

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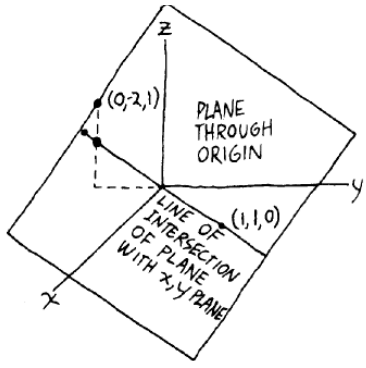

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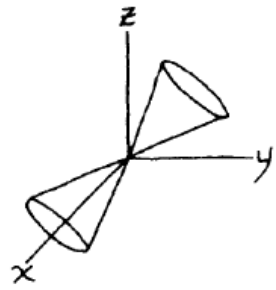

11. $-x+y+\sqrt{3}\,z=5$

12. $x = 0, y = 2, z = 5$

$\rho = \sqrt{29}, \theta = 90, \phi = \tan^{-1}\frac{2}{5}$

13. (a) $x^2 + y^2 = 4$

(b) $r = 2$

(c) $\rho = 2\csc\phi$

14. $\vec{v} = (1,1,-2), \vec{u} \times \vec{v} = (3,9,6)$