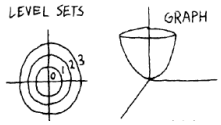
MSE, 미적분학

[연습문제 답안 이용 안내]

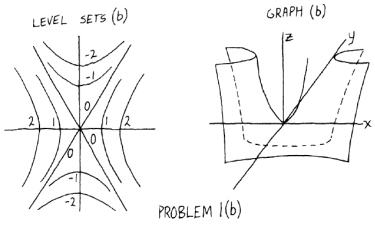
Chapter 11 연습문제 답안

《Section 11.1》

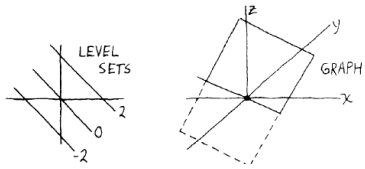
1. (a)



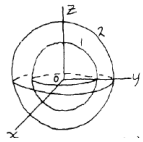
(b)



(c)



2. (a)



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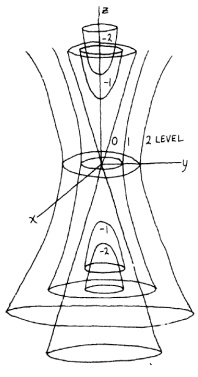

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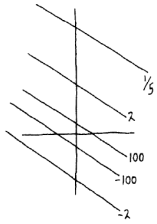
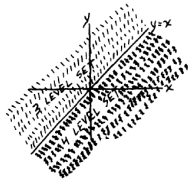


(d)



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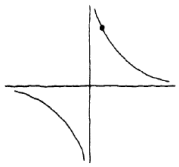


4. (a) $f(x) = \frac{1}{3}(6-2x)$

6레벨 $g(x,y) = 2x+3y$

0레벨 $h(x,y) = 2x+3y-6$

(b) $f(x) = x^2+2y^2$

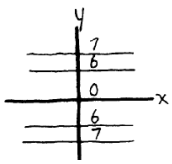
6레벨 $g(x,y,z) = z-x^2-2y^2$

0레벨 $h(x,y) = x^2+2y^2-z$

(c) 0레벨 $f(x,y,z) = z^2+2y^2-x$

(d) 4레벨 $f(x,y,z) = x^2+y^2$

5.



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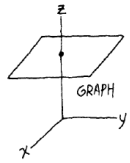
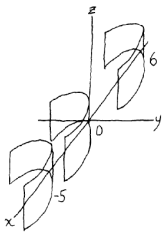


7. (a)



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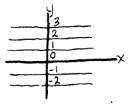

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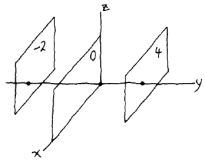
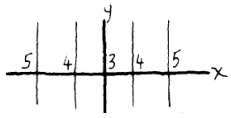


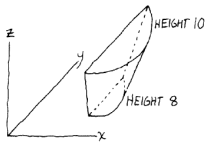
8. $(x-2)^2+(y-1)^2+(z-3)^2=\frac{1}{9}$

9. (a) $f(3,2)$

(b)



10.



11. 아니오

《Section 11.2》

1. (a) $\frac{\delta z}{\delta x} = 2x + 6x^2y^2, \frac{\delta z}{\delta y} = 4x^3y$

   (b) $\frac{\delta z}{\delta x} = e^{-y}, \frac{\delta z}{\delta y} = -xe^{-y}$

   (c) $\frac{\delta z}{\delta x} = \frac{y}{(x+y)^2}, \frac{\delta z}{\delta y} = \frac{-x}{(x+y)^2}$

   (d) $\frac{\delta z}{\delta x} = 8x(2x+5y)^3 + (2x+5y)^4, \frac{\delta z}{\delta y} = 20x(2x+5y)^4$

   (e) $\frac{\delta z}{\delta x} = \frac{-3y}{x^2}, \frac{\delta z}{\delta y} = \frac{3}{x}$

2. (a) $\frac{\delta^2 f}{\delta y^2} = \frac{-9}{(2x+3y)^2}, \frac{\delta^2 f}{\delta x \delta y} = \frac{-6}{(2x+3y)^2}$

   (b) $\frac{\delta^2 f}{\delta y^2} = \frac{-2xy}{(x^2+y^2)^2}, \frac{\delta^2 f}{\delta x \delta y} = \frac{y^2-x^2}{(x^2+y^2)^2}$

   (c) $\frac{\delta^2 f}{\delta y^2} = \frac{4x}{(x-y)^3}, \frac{\delta^2 f}{\delta x \delta y} = \frac{-2x-2y}{(x-y)^3}$

3. (a) g_{cba}

   (b) u_{txx}

4. (a) $\frac{z}{y} cos \frac{x}{y}$

   (b) $-\frac{xz}{y^2} cos \frac{x}{y}$

   (c) $\sin(x/y)$

   (d) $\frac{1}{y} cos \frac{x}{y}$

5. (a) $e^x \sin y$

   (b) $e^x \cos y$

   (c) 0

   (d) $-xcosy$

   (e) $-\sin y$

6. (a) $\delta x/\delta\rho = \sin\phi\cos\theta,\ \delta^2 x/\delta\rho^2 = 0$

(b) $\delta x/\delta\rho = -\rho\sin\phi\sin\theta, \delta^2 x/\delta\phi\,\delta\theta = -\rho\cos\phi\sin\theta$

7. $\delta f/\delta x < 0, \delta f/\delta y < 0$

8. $\delta f/\delta a < 0, \delta f/\delta b > 0$

9. 달리기와 자전거 타기 모두 이익을 발생시킨다.
자전거 타기보다 달리기가 더 많은 이익을 발생시킨다.

10. 오른쪽으로 움직일 때 : -8
아래로 움직일 때 : 12

11. $\delta f/\delta x > 0, \delta f/\delta y = 0$

12. $\delta f/\delta x < 0, \delta f/\delta y = 0$

13. $\delta f/\delta x > 0, \delta f/\delta y < 0, \delta f/\delta z = 0$

14. $\delta g/\delta x > \delta f/\delta x$

《Section 11.3》

1. $$\frac{\delta w}{\delta s}=\frac{\delta w}{\delta x}\frac{\delta x}{\delta s}+\frac{\delta w}{\delta y}\frac{\delta y}{\delta s}+\frac{\delta w}{\delta z}\frac{\delta z}{\delta s}$$

2. $$\frac{du}{dt}=\frac{\delta u}{\delta x}\frac{\delta x}{\delta a}\frac{da}{dt}+\frac{\delta u}{\delta y}\frac{\delta y}{\delta a}\frac{da}{dt}+\frac{\delta u}{\delta z}\frac{\delta z}{\delta a}\frac{da}{dt}+\frac{\delta u}{\delta x}\frac{\delta x}{\delta b}\frac{db}{dt}+\frac{\delta u}{\delta y}\frac{\delta y}{\delta b}\frac{db}{dt}+\frac{\delta u}{\delta z}\frac{\delta z}{\delta b}\frac{db}{dt}$$

3. $$\frac{\delta p}{\delta y}=\frac{dp}{dt}\frac{\delta t}{\delta y}$$

4. (a) $t^2\cos(t^3\ln t)+3t^2\ln t\cos(t^3\ln t)$

   (b) $w=\dfrac{1}{x\sin y}$

   $$\frac{\delta w}{\delta x}=\frac{1}{\sin y}\times(-x^{-2})=-\frac{1}{x^2\sin y}$$

   $$\frac{\delta w}{\delta x}=\frac{\delta w}{\delta u}\frac{\delta u}{\delta x}=-\frac{1}{x^2\sin y}$$

5. (a) $$\frac{\delta w}{\delta t}=\frac{\delta w}{\delta x}\frac{dx}{dt}+\frac{\delta w}{\delta y}\frac{dy}{dt}$$

   (b) $t=3$일 때, 온도가 2도씩 떨어진다.

6. $$\frac{\delta z}{\delta u}=\cos t\frac{\delta z}{\delta x}\frac{\delta t}{\delta u}+6t^2\frac{\delta z}{\delta y}\frac{\delta t}{\delta u}$$

7. $$\frac{\delta u}{\delta x}=\frac{du}{d\rho}\frac{\delta \rho}{\delta x}=\frac{x}{\sqrt{x^2+y^2+z^2}}\frac{du}{d\rho}=\frac{x}{\rho}\frac{du}{d\rho}$$

   $$\frac{\delta u}{\delta y}=\frac{y}{\rho}\frac{du}{d\rho}$$

   $$\frac{\delta u}{\delta z}=\frac{z}{\rho}\frac{du}{d\rho}$$

   $$(\frac{\delta u}{\delta x})^2+(\frac{\delta u}{\delta y})^2+(\frac{\delta u}{\delta z})^2=\frac{x^2+y^2+z^2}{\rho^2}(\frac{du}{d\rho})^2=(\frac{du}{d\rho})^2$$

8. $$u_x=-\frac{1}{x^2}\frac{\delta u}{\delta p}-\frac{1}{x^2}\frac{\delta u}{\delta q},$$

   $$u_y=\frac{1}{y^2}\frac{\delta u}{\delta p}$$

   $$u_z=\frac{1}{z^2}\frac{\delta u}{\delta q},$$

9. $\frac{\delta z}{\delta x} = 2x\frac{dz}{dt}, \frac{\delta z}{\delta y} = 2y\frac{dz}{dt}, y\frac{\delta z}{\delta x} = x\frac{dz}{dt} = 2xy\frac{dz}{dt}$

10. $xu_x + yu_y + zu_z = 2x^2w = 2u$

《Section 11.4》

1. $$\frac{\delta p}{\delta u}=\frac{\delta p}{\delta a}\frac{\delta a}{\delta u}+\frac{\delta p}{\delta b}\frac{\delta b}{\delta u}=3\frac{\delta p}{\delta a}+5\frac{\delta p}{\delta b}$$
$$\frac{\delta^2 p}{\delta u^2}=3\frac{\delta}{\delta u}(\frac{\delta p}{\delta a})+5\frac{\delta}{\delta u}(\frac{\delta p}{\delta b})=9\frac{\delta^2 p}{\delta t}+30\frac{\delta^2 p}{\delta a\delta b}+25\frac{\delta^2 p}{\delta b^2}$$

2. $$\frac{dz}{dt}=\frac{\delta z}{\delta x}\frac{dx}{dt}+\frac{\delta z}{\delta y}\frac{dy}{dt}=3\frac{\delta z}{\delta x}+4\frac{\delta z}{\delta y}$$
$$\frac{d^2 z}{dt^2}=3(3\frac{\delta^2 z}{\delta x^2}+4\frac{\delta^2 z}{\delta y\delta x})+4(3\frac{\delta^2 z}{\delta x\delta y}+4\frac{\delta^2 z}{\delta y^2})$$
$$=9\frac{\delta^2 z}{\delta x^2}+16\frac{\delta^2 z}{\delta y^2}+24\frac{\delta^2 z}{\delta x\delta y}$$

3. $$6\frac{\delta^2 u}{\delta x^2}+2a^3b\frac{\delta^2 u}{\delta y^2}+\frac{\delta^2 u}{\delta x\delta y}(2a^2+6ab)+2a\frac{\delta u}{\delta y}$$

4. $$\frac{dw}{dt}=\frac{\delta w}{\delta x}\frac{dx}{dt}+\frac{\delta w}{\delta y}\frac{dy}{dt}=3t^2\frac{\delta w}{\delta x}+2t\frac{\delta w}{\delta y}$$
$$\frac{d^2 w}{dt^2}=9t^4\frac{\delta^2 w}{\delta x^2}+4t^2\frac{\delta^2 w}{\delta y^2}+12t^3\frac{\delta^2 w}{\delta x\delta y}+2\frac{\delta w}{\delta y}$$

5. (a) $$\frac{\delta r}{\delta x}=\frac{x}{\sqrt{x^2+y^2}}=\frac{x}{r}$$
$$\frac{\delta\theta}{\delta x}=-\frac{y}{r^2}$$

(b) $$\frac{\delta^2\theta}{\delta x^2}=\frac{2xy}{r^4}$$

(c) $$\frac{\delta^2 v}{\delta r^2}+\frac{1}{r}\frac{\delta v}{\delta r}+\frac{1}{r^2}\frac{\delta^2 v}{d\theta^2}$$

6. $$\frac{\delta v}{\delta x}\frac{d^2 x}{dt^2}+\frac{\delta v}{\delta y}\frac{d^2 y}{dt^2}+\frac{\delta^2 v}{\delta x^2}(\frac{dx}{dt})^2+\frac{\delta^2 v}{\delta x^2}(\frac{dy}{dt})^2+2\frac{\delta^2 v}{\delta x\delta y}\frac{dx}{dt}\frac{dy}{dt}$$

7. $$w_{xx}=\frac{}{\delta p^2}+2\frac{\delta^2 w}{\delta p\delta q}+\frac{\delta^2 w}{\delta q^2}$$
$$w_{tt}=-c\frac{\delta}{\delta t}(\frac{\delta w}{\delta p})+c\frac{\delta}{\delta t}(\frac{\delta w}{\delta q})$$

《Section 11.5》

1. (a) $\max : 17, \min : 2$

(b) $\max : 64/5, \min : 0$

(c) $\max : -8, \min : -16$

(d) $\max : 4.75, \min : -\sqrt{2}$

2. (a) $\max : (5,3), \min : (-1/2,0)$

(b) $\max : (\frac{1}{2}, \pm\frac{1}{2}\sqrt{3}), \min : (-1/2,0)$

3. (a) $\max : 10$

(b) $\max : \infty, \min : -\infty$

(c) $\max : \infty, \min : -8$

(d) $\max : \infty, \min : -\infty$

4. $(3,2,1)$

5. (a) 2

(b) 2

6. $(\frac{14}{5}, \frac{11}{5}, \frac{28}{5}), (\frac{4}{5}, \frac{9}{5}, \frac{32}{5})$

7. (a) 높이가 낮고 밑넓이가 넓은 경우
높이가 높고 밑넓이가 작은 경우

(b) $8 \times 8 \times \frac{256}{64}$

(c) $y = 20, x = 4$

《Section 11.6》

1. (a) $14/\sqrt{2}$ (b) $-4/\sqrt{40}$
   (c) -4 (d) $\sqrt{116}$
   (e) $10\cos 157.5 + 4\sin 157.5$

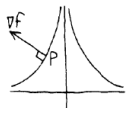
2. $16/\sqrt{2}$

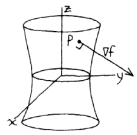
3. (a) $12/\sqrt{29}$ (b) $(-1,2,0), (5,-8,-2)$
   (c) $3/\sqrt{6}$ (d) $-2\vec{i}-\vec{j}-\vec{k},\ \sqrt{6}/m$

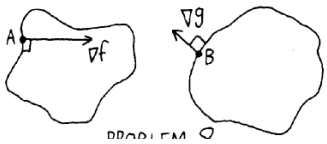
4. 떨어지는 것

5. $4/\sqrt{13}$

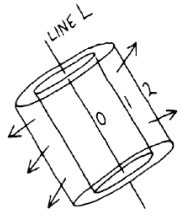
6. $\nabla f = (-4,-2) |\nabla f| = \sqrt{20}$

7. (a) $\nabla f = (2xy, x^2), (-4,1)$
   

   (b) $\nabla f = (2x, 4y, -2z), (2,8,-2)$
   

8. $\nabla f > \nabla g$
   

9. $\nabla f = \vec{0}$

10. $\nabla f = \vec{0}$
    

11. (a) 12 (b) $\sqrt{45}$

(c) $-3/\sqrt{2}$ (d) $x=1+12t, y=2+6t, z=6+2t$

12. (a) -1 (b) $\sqrt{52}$

(c) $2/\sqrt{2}$ (d) $x=1-6t, y=-1-4t, z=1+t$

13. (a) 11 (b) $(1,3,11)$

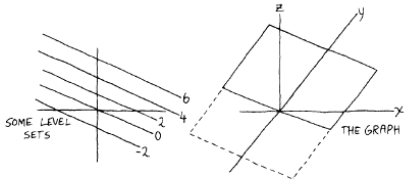
14. (a) $(2,4,12)$ (b) $40/\sqrt{164}$

(c) $(1,-1,-1), (3,2,4), (0,-2,2)$ (d) $(-3,2,4)$

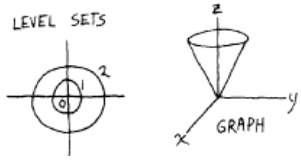
(e) $-\dfrac{1}{2\sqrt{2}}$

《복습문제》

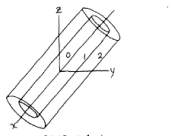
1. (a) $z = 2x + 3y$



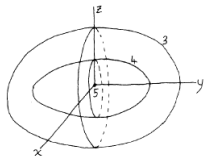
(b) $z^2 = x^2 + y^2$



2. (a) $y = z = 0$



(b) $x^2 + 2y^2 + 3z^2 = 8$



3. (a) y

(b) 0

4. $(\frac{\delta z}{\delta r})^2 + \frac{1}{r^2}(\frac{\delta z}{\delta \theta})^2 = (\frac{\delta z}{\delta x})^2(\cos^2\theta + \sin^2\theta) + (\frac{\delta z}{\delta y})^2(\cos^2\theta + \sin^2\theta)$

5. $8\frac{\delta^2 u}{\delta x \delta y} + 4c\frac{\delta^2 u}{\delta z \delta y} + 2a\frac{\delta^2 u}{\delta x \delta z} + ac\frac{\delta^2 u}{\delta z^2} + \frac{\delta u}{\delta z}$

6. $\max : 0, \min : -12$

7. $\|\nabla f\| = \sqrt{64 - 28x^2}\,(-1 \le x \le 1)$

8. 출발할 때 : $\frac{25}{\sqrt{14}}$, 도착할 때 : $\frac{1}{\sqrt{30}}$

9. (a) $\nabla T = (2x, -1)$
   (b) $-4\vec{i} - \vec{j}$

10. (a) $z = x^2 - y$
    (b) $(-4, -1, -1)$
    (c) $3/\sqrt{2}$
    (d) $\nabla z = -4\vec{i} - \vec{j}$

11. $(2\pi h + 4\pi r)dr = 2\pi r dh$

12. $y = x + \frac{1}{x}$