MSE, 미적분학

[연습문제 답안 이용 안내]

Chapter 07 연습문제 답안

《Section 7.2》

1. $u=x^2,\ du=2xdx$
$\int xe^{x^2}dx=\frac{1}{2}\int e^u du=\frac{1}{2}e^u+C=\frac{1}{2}e^{x^2}+C$

2. $u=3x^2+7,\ du=6xdx$
$\frac{1}{6}\int\sqrt{u}\,du=\frac{1}{6}u^{3/2}/\frac{3}{2}+C=\frac{1}{9}(3x^2+7)^{3/2}+C$

3. $u=4+5x,\ du=5dx$
$\frac{1}{5}\int\sqrt{u}\,du=\frac{1}{5}u^{3/2}/\frac{3}{2}+C=\frac{2}{15}(3+5x)^{3/2}+C$

4. $u=3+7x,\ du=7dx$
$\frac{1}{7}\int 1/\sqrt{u}\,du=\frac{2}{7}\sqrt{u}+C=\frac{2}{7}\sqrt{3+7x}+C$

5. $u=\tan x,\ du=\sec^2 dx$
$\int u^{15}du=\frac{1}{15}u^{15}+C=\frac{1}{15}tan^{15}x+C$

6. $u=x+1,\ du=dx,\ x=u-1$
$\int\frac{u-2}{u^5}du=\int(u^{-4}-2u^{-5})du=\frac{u^{-3}}{-3}-2\frac{u^{-4}}{-4}+C=\frac{1}{3(x+1)^3}+\frac{1}{2(x+1)^4}+C$

7. $u=1+2\sec\theta,\ du=2\sec\theta\tan\theta d\theta$
$\int=\frac{1}{2}\int 1/\sqrt{u}\,du=\frac{1}{2}u^{1/2}/\frac{1}{2}+C=\sqrt{1+2\sec\theta}+C$

8. $u=\ln x,\ du=1/xdx$
$\int 1/udu=\ln|u|+C=\ln|\ln x|+C$

9. $u=x^2,\ du=2xdx$
$\int x^3\sin udu/2x=\frac{1}{2}\int x^2\sin udu=\frac{1}{2}\int u\sin udu=\frac{1}{2}(\sin x^2-x^2\cos x^2)+C$

10. $u=1+3x,\ du=3dx$
$\frac{1}{3}\int u^7du=\frac{1}{24}u^8+C=\frac{1}{24}(1+3x)^8+C$

11. $u=2-3x,\ du=-xdx$
$-\frac{1}{3}\int 1/udu=-\frac{1}{3}\ln|u|+C=-\frac{1}{3}\ln|2-3x|+C$

12. $u=2-x,\ du=-dx$
$-\int\frac{1}{u^3}du=\frac{1}{2u^2}+C=\frac{1}{2(2-x)^2}+C$

13. $u=\frac{1}{2}\theta-1,\ du=\frac{1}{2}d\theta$
$2\int\cos udu=2\sin u+C=2\sin(\frac{1}{2}\theta-1)+C$

14. $u=-x,\ du=-dx$
$\int(-u)e^u\cdot -du=\int ue^udu=e^u(u-1)+C=e^{-x}(-x-1)+C$

15. $u=\cos x,\ du=-\sin xdx$
$-\int u^3du=-\frac{1}{4}u^4+C=-\frac{1}{4}\cos^4x+C$

16. $u=-x,\ du=-dx$
$-\int e^udu=-e^u+C=-e^{-x}+C$

17. $u=3x,\ du=3dx$
$\int\frac{1}{3}\sin u\cdot\frac{1}{3}du=\frac{1}{9}\int u\sin udu=\frac{1}{9}(\sin u-u\cos u)+C$
$=\frac{1}{9}(\sin 3x-3xcos3x)+C$

18. $u=\pi x,\ du=\pi dx$
$(1/\pi)\int\sin^2udu=(1/\pi)\frac{1}{2}(u-\sin u\cos u)+C=\frac{1}{2\pi}(\pi x-\sin\pi xcos\pi x)+C$

19. $3\int x\sin xdx=3(\sin x-xcosx)+C$

20. $u = 3x, du = 3x$

$$\int (\frac{1}{3}u)^2 \cos u \cdot \frac{1}{3}du = \frac{1}{27}\int u^2 \cos u du$$

$$= \frac{1}{27}[(u^2-2)\sin u + 2u\cos u] + C = \frac{1}{27}(9x^2-2)\sin 3x + \frac{6}{27}x cos 3x + C$$

21. $u = 2x+3, du = 2dx$

$$\frac{1}{2}\int \ln u du = \frac{1}{2}(u\ln u - u) + C = \frac{1}{2}(2x+3)\ln(2x+3) - \frac{1}{2}(2x+3) + C$$

22. $\int \sec x dx = \ln|\sec x + \tan x| + C$

23. $u = \sqrt{3}x, du = \sqrt{3}dx$

$$\int \frac{1}{1+(\sqrt{3}x)^2}dx = \int \frac{1}{1+u^2}\frac{du}{\sqrt{3}} = \frac{1}{\sqrt{3}}tan^{-1}\sqrt{3}x + C$$

24. (a) $u = 3x, du = 3dx$

$$\frac{1}{3}\int \tan^{-1}u du = \frac{1}{3}(u\tan^{-1}u - \frac{1}{2}\ln(1+u^2)) + C$$

$$= x\tan^{-1}3x - \frac{1}{6}ln(1+9x^2) + C$$

(b) 아직 연산할 수 없다.

25. $u = \cos x, du = -\sin x dx$

$$-\int 1/u du = -\ln|u| + C = -\ln|\cos x| + C$$

26. $\int \sec x dx = \int \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x}dx = \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}dx$

$u = \sec x + \tan x, du = (\sec x \tan x + \sec^2 x)dx$

$$\int du/u = \ln|u| + C = \ln|\sec x + \tan x| + C$$

27. $\int \sin^2 x dx = \frac{1}{2}\int (1-\cos 2x)dx = \frac{1}{2}\int dx - \frac{1}{2}\int \cos 2x dx$

$$= \frac{1}{2}x - \frac{1}{4}\sin 2x + C$$

《Section 7.3》

1. $2+6x-x^2=-(x^2-6x-2)=-([x-3]^2-11)=11-(x-3)^2$
$\int 1/\sqrt{11-(x-3)^2}\,dx$
$u=x-3,\ du=dx$
$\int \frac{du}{\sqrt{11-u^2}}=\sin^{-1}\frac{u}{\sqrt{11}}+C=\sin^{-1}\frac{x-3}{\sqrt{11}}+C$

2. $x+2x^2=2(x^2+\frac{1}{2}x+\frac{1}{16}-\frac{1}{16})=2([x+\frac{1}{4}]^2-\frac{1}{16})$
$u=x+\frac{1}{4},\ du=dx$
$\frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{(x+\frac{1}{4})^2-\frac{1}{16}}}=\frac{1}{\sqrt{2}}\int \frac{du}{\sqrt{u^2-\frac{1}{16}}}$
$=\frac{1}{\sqrt{2}}\ln|x+\frac{1}{4}+\sqrt{(x+\frac{1}{4})^2-\frac{1}{16}}|+C$

3. $\frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{x^2-\frac{5}{3}}}=\frac{1}{\sqrt{3}}\ln|x+\sqrt{x^2-\frac{5}{3}}|+C$

4. $\int (x^2-4+\frac{2x+16}{x^2+4})dx=\frac{x^3}{3}-4x+2\int \frac{x}{x^2+4}dx+16\int \frac{1}{x^2+4}dx$
$=\frac{1}{3}x^3-4x+\ln(x^2+4)+8\tan^{-1}\frac{1}{2}x+C$

5. $\int x\sqrt{(x+1)^2-1}\,dx$
$u=x+1,\ du=dx$
$\int (u-1)\sqrt{u^2-1}\,du=\int u\sqrt{u^2-1}\,du-\int \sqrt{u^2-1}\,du$
$=\frac{1}{3}(u^2-1)^{3/2}-\frac{1}{2}u\sqrt{u^2-1}+\frac{1}{2}\ln|u+\sqrt{u^2-1}|+C$
$=\frac{1}{3}(x^2+2x)^{3/2}-\frac{1}{2}(x+1)\sqrt{x^2+2x}+\frac{1}{2}ln|x+1+\sqrt{x^2+2x}|+C$

6. (a) $\int (\frac{1}{2}-\frac{3}{2x+6})dx=\frac{1}{2}x-\frac{3}{2}ln|2x-6|+C$

(b) $a=6,\ b=2$
$\frac{3}{2}+\frac{1}{2}x-\frac{3}{2}\ln|2x+6|+C=\frac{1}{2}x-\frac{3}{2}\ln|2x-6|+K$

(c) $u = 2x + 6,\ du = 2dx$

$$\int \frac{\frac{1}{2}(u-6)}{u}\frac{du}{2} = \frac{1}{4}\int (1 - \frac{6}{u})du = \frac{1}{4}u - \frac{3}{2}\ln|u| + C$$

$$= \frac{1}{4}u - \frac{3}{2}\ln|u| + C = \frac{1}{4}(2x+6) - \frac{3}{2}\ln|2x+6| + C$$

7. $\int \left(1 - \frac{1}{x^2+1}\right)dx = x - \tan^{-1}x + C$

《Section 7.4》

1. (a) $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x^3}+\dfrac{D}{x+1}+\dfrac{E}{2x+3}$

(b) $\dfrac{A}{x-(-1+\sqrt{3})}+\dfrac{B}{x-(-1-\sqrt{3})}+\dfrac{Cx+D}{x^2-2x+2}$

2. (a) $\dfrac{12}{(x-\sqrt{3})(x+\sqrt{3})}=\dfrac{A}{x+\sqrt{3}}+\dfrac{B}{x-\sqrt{3}}$

$12=A(x-\sqrt{3})+B(x+\sqrt{3})$

$x=\sqrt{3}$이라고 하면

$12=2\sqrt{3}B$

$B=\dfrac{6}{\sqrt{3}}=2\sqrt{3}$

$x=-\sqrt{3}$이라고 하면

$12=-2\sqrt{3}A$

$A=-2\sqrt{3}$

$\therefore \dfrac{-2\sqrt{3}}{x+\sqrt{3}}+\dfrac{2\sqrt{3}}{x-\sqrt{3}}$

(b) $\dfrac{1}{(x-4)(2x+3)}=\dfrac{A}{x-4}+\dfrac{B}{2x+3}$

$1=A(2x+3)+B(x-4)$

$x=4$라고 하면

$1=11A,\ A=1/11$

$x=-3/2$라고 하면

$1=-\dfrac{11}{2}B,\ B=-2/11$

$\therefore \dfrac{1/11}{x-4}-\dfrac{2/11}{2x+3}$

(c) $\dfrac{5x}{(x^2+1)(x-2)}=\dfrac{Ax+B}{x^2+1}+\dfrac{C}{x-2}$

$5x=(Ax+B)(x-2)+C(x^2+1)$

$x=2$라면 $10=5C,\ C=2$

$0=A+C,\ A=-2$

$5=B-2A,\ B=1$

$\therefore \dfrac{-2x+1}{x^2+1}+\dfrac{2}{x-2}$

(d) $\dfrac{2x+3}{(x-2)^2}=\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}$

$2x+3=A(x-2)+B$

$x=2$라고 하면 $7=B,\ 2=A$

$\therefore \dfrac{2}{x-2}+\dfrac{7}{(x-2)^2}$

3. (a) $\dfrac{3}{(2-x)(x+1)}=\dfrac{A}{2-x}+\dfrac{B}{x+1}$

$3=A(x+1)+B(2-x)$

$x=2$라고 하면 $3=3A,\ A=1$

$x=-1$이라고 하면 $3=3B,\ B=1$

$\therefore -\ln|2-x|+\ln|x+1|+C$

(b) $\displaystyle\int\frac{3dx}{-x^2+x+2}$에 $a=-1, b=1, c=2$를 대입하면

$\ln\left|\dfrac{-2x+1-3}{-2x+1+3}\right|+C$

$=\ln\left|\dfrac{-x-1}{2-x}\right|+C=\ln\dfrac{|x+1|}{|2-x|}+C$

$=\ln|x+1|-\ln|2-x|+C$

4. (a) $2\displaystyle\int\frac{xdx}{x^2-4x+4}+3\int\frac{dx}{x^2-4x+4}=\frac{2}{x-2}-\frac{7}{(x-2)^2}$

$=2\ln|x-1|-\ln|2-x|+C$

(b) $\dfrac{8x}{(x^2-1)(x^2+1)}=\dfrac{8x}{(x+1)(x-1)(x^2+1)}=\dfrac{A}{x+1}+\dfrac{B}{x-1}+\dfrac{Cx+D}{x^2+1}$

$8x=A(x-1)(x^2+1)+B(x+1)(x^2+1)+(Cx+D)(x^2-1)$

$x=1$이라고 하면 $8=4B,\ B=2$

$x=-1$이라고 하면 $-8=-4A,\ A=2$

$A+B+C=0,\ C=-4$

$-A+B+D=0,\ D=0$

$\dfrac{2}{x+1}+\dfrac{2}{x-1}-\dfrac{4x}{x^2+1}$

$\therefore 2\ln|x+1|+2\ln|x-1|-2\ln(x^2+1)+C$

(c) $\dfrac{-2/9}{x}-\dfrac{1/3}{x^2}+\dfrac{4/9}{2x-3}$

$\therefore -\dfrac{2}{9}\ln|x|+1/3x+\dfrac{2}{9}\ln|2x-3|+C$

5. $\dfrac{1}{x(a+bx)^2}=\dfrac{A}{x}+\dfrac{B}{a+bx}+\dfrac{C}{(a+bx)^2}$

$1=A(a+bx)^2+Bx(a+bx)+Cx$

$x=0$이라면 $1=a^2A,\ A=1/a^2$

$x=-a/b$라면 $1=-(a/b)C,\ C=-b/a$

$0=b^2A+bB,\ B=-b/a^2$

$\dfrac{1}{a^2}\displaystyle\int\frac{1}{x}dx-\frac{b}{a^2}\int\frac{1}{a+bx}dx-\frac{b}{a}\int\frac{1}{(a+bx)^2}dx$

$u=a+bx$로 두고 계산하면

$\dfrac{1}{a^2}\ln|x|-\dfrac{1}{a^2}\ln|a+bx|+\dfrac{1}{a}\dfrac{1}{a+bx}+C$

$=-\dfrac{1}{a^2}(\ln|a+bx|-\ln|x|)+\dfrac{1}{a(a+bx)}+C$

6. $\dfrac{x^2}{x^2+5x+4}=1-\dfrac{5x+4}{x^2+5x+4}$

$\displaystyle\int 1-\frac{5x+4}{x^2+5x+4}=\int dx-5\int\frac{xdx}{x^2+5x+4}-4\int\frac{dx}{x^2+5x+4}$

$\displaystyle-5\cdot\frac{1}{2}\ln|x^2+5x+4|+\frac{17}{2}\int\frac{dx}{x^2+5x+4}$

$\displaystyle=x-\frac{5}{2}\ln|x^2+5x+4|+\frac{17}{2}\frac{1}{3}\ln|\frac{2x+5-3}{2x+5+3}|$

$\displaystyle=x-\frac{5}{2}\ln|x^2+5x+4|+\frac{17}{6}\ln|\frac{x+1}{x+4}|$

$\displaystyle=x-\frac{5}{2}\ln|x+4|\ln|x+1|+\frac{17}{6}\ln|\frac{x+1}{x+4}|$

$\displaystyle=x-\frac{5}{2}\ln|x+4|-\frac{5}{2}\ln|x+1|+\frac{17}{6}\ln|x+1|-\frac{17}{6}\ln|x+4|$

$\displaystyle=x+\frac{1}{3}\ln|x+1|-\frac{16}{3}\ln|x+4|$

《Section 7.5》

1. (a) $u=x, dv=e^{x}dx$라고 하면,
$du=dx, v=e^{x}$
$\int xe^{x}dx=xe^{x}-\int e^{x}dx=xe^{x}-e^{x}+C$

(b) $u=\tan^{-1}x, dv=dx$
$du=dx/(1+x^{2}), v=x,$
$\int \tan^{-1}dx=x\tan^{-1}x-\int \frac{x}{1+x^{2}}dx$
$=x\tan^{-1}x-\frac{1}{2}ln(1+x^{2})+C$

(c) $u=\sin^{-1}x, dv=dx$라고 하자.
$du=dx/\sqrt{1-x^{2}}, v=x$
$\int \sin^{-1}xdx=x\sin^{-1}x-\int \frac{x}{\sqrt{1-x^{2}}}dx=x\sin^{-1}x+\sqrt{1-x^{2}}+C$

2. (a) $u=\cos(\ln x), dv=dx, v=x$라고 하자.
$\int \cos(\ln x)dx=x cos(\ln x)+\int \sin(\ln x)dx$
$u=\sin(\ln x), dv=dx$라고 하자.
$du=\cos(\ln x)\cdot(1/x)dx, v=x,$
$\int \cos(\ln x)dx=x\cos(\ln x)+x\sin(\ln x)-\int \cos(\ln x)dx$
$2\int \cos(\ln x)dx=x\cos(\ln x)+x\sin(\ln x)$
$\int \cos(\ln x)dx=\frac{1}{2}x[\cos(\ln x)+x sin(\ln x)]+C$

(b) $u=x^{2}, dv=e^{x}dx$라고 하자.
$du=2xdx, v=e^{x}, \int x^{2}e^{x}dx=x^{2}e^{x}-2\int xe^{x}dx$
$u=x, dv=e^{x}dx$
$du=dx, v=e^{x},$
$\int x^{2}e^{x}dx=x^{2}e^{x}-2(xe^{x}-\int e^{x}dx)=x^{2}e^{x}-2xe^{x}+2e^{x}+C$

(c) $u=\tan^{-1}x, dv=xdx$라고 하자.
$du=dx/(1+x^{2}), v=\frac{1}{2}x^{2}$
$\int x\tan^{-1}xdx=\frac{1}{2}x^{2}\tan^{-1}x-\frac{1}{2}\int \frac{x^{2}}{1+x^{2}}dx$
$=\frac{1}{2}x^{2}\tan^{-1}x-\frac{1}{2}\int \left(1-\frac{1}{1+x^{2}}\right)dx$
$=\frac{1}{2}x^{2}\tan^{-1}x-\frac{1}{2}x+\frac{1}{2}\tan^{-1}x+C$

3. $u=\sec x,\ dv=\sec^2 x dx$라 하자.
$du=\sec x\tan x dx,\ v=\tan x,$
$\int \sec^3 x dx=\sec x\tan x-\int \sec x\tan^2 x dx$
$=\sec x\tan x-\int \sec x\cdot(\sec^2 x-1)dx$
$=\sec x\tan x-\int \sec^3 x dx+\int \sec x dx$
$=\sec x\tan x-\int \sec^3 x dx+\ln|\sec x+\tan x|$
따라서 $2\int \sec^3 x dx=\sec x\tan x+\ln|\sec x+\tan x|,$
$\int \sec^3 x dx=\frac{1}{2}(\sec x\tan x+\ln|\sec x+\tan x|)+C$
$\frac{1}{2}(\sec x tan x+\ln|\sec x+\tan x|)+C$

4. $u=x,\ dv=xe^{-x^2}dx$라고 하자.
$du=dx,\ v=\int xe^{-x^2}dx=-\frac{1}{2}e^{-x^2}(u=-x^2,\ du=-2xdx)$
$\int x^2e^{-x^2}dx=-\frac{1}{2}xe^{-x^2}+\frac{1}{2}\int e^{-x^2}dx=-\frac{1}{2}xe^{-x^2}+\frac{1}{2}Q(x)+C$

《Section 7.6》

1. $u=x^n, dv=e^x dx$라고 하자.
$du=nx^{n-1}dx, v=e^x, \int x^n e^x dx = x^n e^x - n\int x^{n-1}e^x dx$

2. $\int (\sec^2 x-1)\tan^{n-2}x dx = \int \sec^2 x \tan^{n-2}x dx - \int \tan^{n-2}x dx$
첫 적분에서 $u=\tan x, du=\sec^2 x dx$로부터 $\frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x dx$

3. $u=(\ln x)^n, dv=dx$라고 하자.
$du=n(\ln x)^{n-1}\cdot\frac{1}{x}dx, v=x, \int (\ln x)^n dx = x(\ln x)^n - n\int (\ln x)^{n-1}dx$
이를 이용하면, $\int (\ln x)^3 dx = x(\ln x)^4 - 3\int (\ln x)^2 dx$
$= x(\ln x)^3 - 3\left[x(\ln x)^2 - 2\int \ln x dx\right]$
$= x(\ln x)^3 - 3x(\ln x)^2 + 6(x\ln x - x) + C$

4. $\int \sin^m x\cos^n x dx = -\frac{\sin^{m-1}x cos^{n+1}x}{m+n} + \frac{m-1}{m+n}\int \sin^{m-2}x\cos^n x dx$
$\int \sin^m x\cos^n x dx = -\frac{\sin^{m-1}x\cos^{n+1}x}{m+n} + \frac{m-1}{m+n}$
$\cdot\left[\frac{\sin^{m-1}x\cos^{n-1}x}{m-2+n} + \frac{n-1}{m-2+n}\int \sin^{m-2}x\cos^{n-2}x dx\right]$

5. 새로운적분에는분자의x가본래식과다르므로

6. (a) $u=\sin x, du=\cos x dx$라고 하자.
$\int \sin x\cos x dx = \int u du = \frac{1}{2}u^2 + C = \frac{1}{2}\sin^2 x + C$
(b) $u=\cos x, du=-\sin x dx$라고 하자.
$-\int u^{12}du = -\frac{1}{13}u^{13} + C = -\frac{1}{13}\cos^{13}x + C$
(c)공식 52(c)에 $m=0, n=-5$를 대입하면 $\frac{-\sin x\cos^{-4}x}{-4} + \frac{-3}{-4}\int \sec^3 x dx$
공식 43을 이용하면 $\frac{1}{4}\sin x\sec^4 x + \frac{3}{4}[\frac{1}{2}secxtanx + \frac{1}{2}ln|\sec x + \tan x|] + C$
(d)공식 53을 이용하면 $\int \tan^4 x dx = \frac{1}{3}\tan^3 x - \int \tan^2 x dx$
$= \frac{1}{3}\tan^3 x - (\tan x - x) + C$

(e) $u=\sin x,\ du=\cos x dx$라고 하자.

$$\int du/u^2 = -1/u + C = -1/\sin x + C$$

(f) $$-\frac{\sin^3 x\cos^{-2}x}{-2} - \frac{1}{2}\int \frac{\sin^2 x}{\cos x}dx$$
$$= \frac{1}{2}\sin^3 x\sec^2 x - \frac{1}{2}(-\sin x + \int \sec x dx) + C$$
$$= \frac{1}{2}\sin^3 x\sec^2 x - \frac{1}{2}(-\sin x + \ln|\sec x + \tan x|) + C$$

(g) $$\int \sin^4 x(1-\sin^2 x)\cos x dx = \int (u^4 - u^6)du$$
$$= \frac{1}{5}u^5 - \frac{1}{7}u^7 + C = \frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C$$

(h) $u=3x,\ du=3dx$라고 하자.

$$\frac{1}{3}\int \sin^4 u du$$
$$= \frac{1}{3}(-\frac{1}{4}\sin^3 u\cos u + \frac{3}{4}\int \sin^2 u du$$
$$= -\frac{1}{12}\sin^3 u\cos u + \frac{1}{4}\cdot\frac{1}{2}(u - \sin u\cos u) + C$$
$$= -\frac{1}{12}\sin^3 3x\cos 3x + \frac{1}{8}(3x - \sin 3x\cos 3x) + C$$

7. $$\frac{-\sin^2 x\cos^{99}x}{101} - \frac{2}{101(99)}\cos^{99}x$$
$$= \frac{-\cos^{99}x}{99}\left[\frac{99}{101}\sin^2 x + \frac{2}{101}\right]$$
$$= -\frac{\cos^{99}x}{99}[1 - \frac{99}{101}\cos^2 x]$$
$$= -\frac{\cos^{99}x}{99} + \frac{\cos^{101}x}{101}$$

《Section 7.7》

1. (a) $\sqrt{x^2-a^2}=a\tan u,\ x=a\sec u,$

$dx=a\sec u\tan u du$

$\int \frac{1}{a\tan u} a\sec u\tan u du$

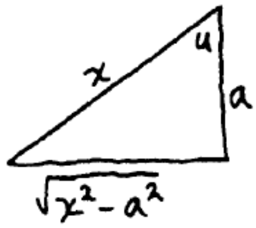
$=\int \sec u du = \ln|\sec u+\tan u|+C$

$=\ln\left|\frac{x}{a}+\frac{\sqrt{x^2-a^2}}{a}\right|+C$

$=\ln\frac{|x+\sqrt{x^2-a^2}|}{a}+C$

$=\ln|x+\sqrt{x^2-a^2}|-\ln a+C$

$=\ln|x+\sqrt{x^2-a^2}|+K$



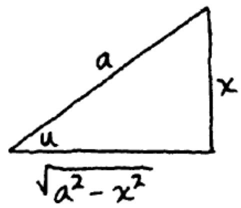
(b) $\sqrt{a^2-x^2}=a\cos u,\ x=a\sin u,\ dx=a\cos u du$

$\int a\cos u \cdot a\cos u du$

$=a^2\int \cos^2 u du = \frac{1}{2}a^2(u+\sin u\cos u)+C$

$=\frac{1}{2}a^2(\arcsin\frac{x}{a}+\frac{x}{a}\frac{\sqrt{a^2-x^2}}{a})+C$

$=\frac{1}{2}a^2\arcsin x/a+\frac{1}{2}x\sqrt{a^2-x^2}+C$



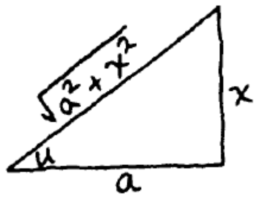
(c) $\sqrt{a^2+x^2}=a\sec u,\ x=a\tan u,\ dx=a\sec^2 u du$

$\int \frac{a\sec u}{a\tan u} a\sec^2 u du = a\int \frac{du}{\cos^2 u\sin u}$

$=a(\sec u+\int \csc u du)$

$=a\sec u - a\ln|\csc u+\cot u|+C$

$=\sqrt{a^2+x^2}-a\ln|\frac{\sqrt{a^2+x^2}}{x}+\frac{a}{x}|+C$

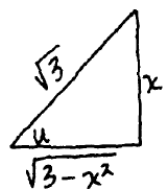


2. $\sqrt{3-x^2}=\sqrt{3}\cos u,\ x=\sqrt{3}\sin u,\ dx=\sqrt{3}\cos u du$

$\int \frac{\sqrt{3}\cos u}{3\sin^2 u}\sqrt{3}\cos u du = \int \cot^2 u du$

$=-\cot u-u+C$

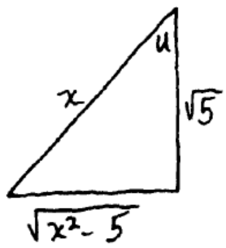
$=-\frac{\sqrt{3-x^2}}{x}-\sin^{-1}\frac{x}{\sqrt{3}}+C$



3. $\sqrt{x^2-5}=\sqrt{5}\tan u,\ x=\sqrt{5}\sec u,\ dx=\sqrt{5}\sec u\tan u du$

$\int \frac{\sqrt{5}\sec u\tan u du}{5\sec^2 u\sqrt{5}\tan u}=\frac{1}{5}\int \cos u du$

$=\frac{1}{5}\sin u+C=\frac{1}{5}\frac{\sqrt{x^2-5}}{x}+C$

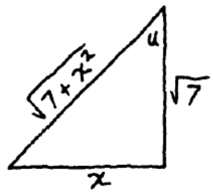


4. $\sqrt{7+x^2} = \sqrt{7}\sec u$, $(7+x^2)^2 = (\sqrt{7}\sec u)^4 = 49\sec^4 u$,
$x = \sqrt{7}\tan u$, $dx = \sqrt{7}\sec^2 u du$

$$\int \frac{\sqrt{7}\sec^2 u du}{49\sec^4 u} = \frac{1}{7\sqrt{7}}\int \cos^2 u du$$

$$= \frac{1}{7\sqrt{7}}\frac{1}{2}(u + \sin u \cos u) + C$$

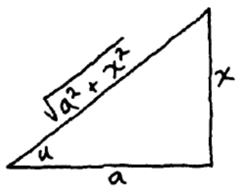
$$= \frac{1}{14\sqrt{7}}(\tan^{-1}\frac{x}{\sqrt{7}} + \frac{x\sqrt{7}}{7+x^2}) + C$$



5. $x = a\tan u$, $dx = a\sec^2 u du$, $\sqrt{a^2+x^2} = a\sec u$

$$\int \frac{a\sec^2 u du}{(a\sec u)^3} = \frac{1}{a^2}\int \cos u\, du = \frac{1}{a^2}\sin u + C$$

$$= \frac{x}{a^2\sqrt{a^2+x^2}} + C$$



《Section 7.8》

1. $u = \sqrt{x}$

2. $u = 1 - x^2$

3. $a = \sqrt{3}$

4. $u = 2x + 3$

5. $u = x, dv = (x-1)^{20}dx$

6. $-e^{-x} + C$

7. 긴 나눗셈

8. $u = 4 - x^2$

9. $x^2 + 9x + C$

10. 공식 19

11. 공식 9

12. $u = 3x$ 적용 후 공식 31

13. 공식 11

14. $u = \sqrt{2}x$ 적용 후, 공식 21

15. 부분적분

16. 긴 나눗셈

17. 공식 52(b) 적용 후, 공식 42 적용. $\cos^4 x = (1 - \sin^2 x)^2$

18. $u = \pi x$

19. $u = 3x + 1$

20. $u = 9 + 4x^3$

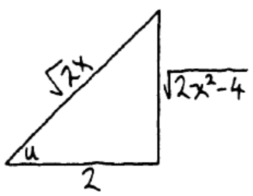
21. $\tan x - \sin x cos x$

22. $u = 1 - x^2$

23. $u = 9 + 4x$

24. $\int \sec^2 x dx$

25.



26. $\sin^4 x = (1 - \cos^2 x)^2, u = \cos x$

27. $x + 4/x + C$

28. 공식 43

29. $u = 2x + 1$

30. $u = x^2$

31. 공식 52(c)

32. 공식 50

33. $u = \sqrt{3}\,x$

34. $u = 3x$

35. $\frac{1}{2}x-\frac{3}{4}ln|2x+3|+C$

36. $u=5x$

37. $u=2-r^2$

38. 공식 42

39. $u=\cos x$

40. $2x+C$

41. $u=2x$

42. $2x+3\ln|x|+C$

43. 공식 46

44. $-\frac{1}{2}\pi\cos(2x/\pi)$

45. $u=\cos 2x$

46. $\frac{1}{5}ln|5x-2|+C$

47. 긴 나눗셈

48. $u=x^2+7$

49. $u=\cos x$

50. $\frac{1}{2}x^2\sin^{-1}x-\frac{1}{2}\int\frac{x^2dx}{\sqrt{1-x^2}}$

51. $u = x^2$

52. 부분 적분

53. 공식 61

54. $u = 2x + 3$

55. $u = x, dv = (x-1)^{20}dx$

56. $-e^{-x} + C$

57. $-\frac{2}{3}(3-x)^{3/2}$

58. $-\frac{3}{5}(2-\frac{1}{3}x)^5$

59. $u = e^x, du = e^x dx$

60. $u = \cos x$

61. $u = \sqrt{3}x$

62. 공식 24

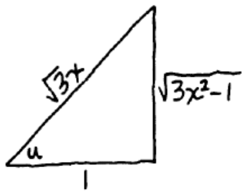
63. $1(b)$

64. $u = 4x + 5$

65. $u = \frac{1}{2}\theta$

66. $u = 1 + x^2$

67.



68. $52(a)$

69. $52(b)$

70. $u = 2x$

71. $x + 2e^x + \frac{1}{2}e^{2x}$

72. $u = \cos x$

73. 완전 제곱식

74. $u = \cos 2x$

75. 부분적분

76. $u = 2 + 3x$

77. 공식 64

78. $\int \frac{x\,dx}{2x^2+x+1} + \int \frac{4\,dx}{2x^2+x-1}$

79. $u = (\ln x)^3$

80. $u = 3x$

81. $u = x^3$

82. $u = 2 + \cos x$

83. 공식 40

84. $\int \cos x dx - \int \cos x sin^2 x dx$

85. $u = \cos x$

《Section 7.9》

1. (a) $du = 3dx,\ \int_2^5 \sin^5 dx = \frac{1}{3}\int_6^{15} \sin^5 \frac{1}{3} u du$

(b) $du = \frac{1}{x}dx,\ \int_1^{e^3} \sin(\ln x)dx = \int_0^3 \sin u \cdot x du = \int_0^3 e^u \sin u du$

(c) $x = 2\csc u,\ dx = -2\csc u \cot u du,\ \sqrt{x^2-4} = 2\cot u$
$x = 2$이면 $u = \pi/2$
$x = 4$이면 $u = \pi/6$
$\int_2^4 \frac{\sqrt{x^2-4}}{x^2}dx = \int_{\pi/2}^{\pi/6} \frac{2\cot u}{4\csc^2 u} \cdot -2\csc u \cot u du = -\int_{\pi/2}^{\pi/6} \frac{\cos^2 u}{\sin u} du$

2. (a) $u = 3x^2 - 1,\ du = 6xdx$라고 하자
$\frac{1}{6}\int_{11}^{47} u^{10} du = \frac{1}{6}\frac{1}{11}u^{11}|_{11}^{47} = \frac{1}{66}(47^{11} - 11^{11})$

(b) $u = e^{-x},\ dv = \cos x dx$
$\int_0^\infty e^{-x}\cos x dx = e^{-x}\sin x|_0^\infty + \int_0^\infty e^{-x}\sin x dx = \int_0^\infty e^{-x}\sin x dx$
$= -e^{-x}\cos x|_0^\infty - \int_0^\infty e^{-x}\cos x dx$
$2\int_0^\infty e^{-x}\cos x dx = 1,\ \int_0^\infty e^{-x}\cos x dx = \frac{1}{2}$

(c) $u = \ln x,\ du = \frac{1}{x}dx$라고 하자
$\int_0^1 u^5 du = \frac{1}{6}u^6|_0^1 = \frac{1}{6}$

(d) $-\frac{1}{4}\sin x \cos^3 x|_{\pi/2}^{\pi} + \frac{1}{4}\int_{\pi/2}^{\pi} \cos^2 x dx$
$= \frac{1}{4} \cdot \frac{1}{2}(x + \sin x \cos x)|_{\pi/2}^{\pi} = \pi/16$

(e) $u = x^3,\ du = 3x^2 dx$
$\int_{-\infty}^2 x^2 e^{x^3} dx = \frac{1}{3}\int_{-\infty}^8 e^u du = \frac{1}{3}e^8$

(f) $u = x^2 + 4,\ du = 2xdx$라고 하자
$\frac{1}{2}\int_4^8 \sqrt{u}\,du = \frac{1}{3}u^{3/2}|_4^8 = 8\sqrt{8}/3 - 8/3$

3. (a) $u=1-x,\ du=-dx$라고 하자.

$$\int_0^1 x^m(1-x)^n dx = -\int_1^0 (1-u)^m u^n du$$
$$= \int_0^1 (1-u)^m u^n du$$
$$= \int_0^1 (1-x)^m x^n dx$$

(b) $u=x+20,\ du=dx$

$$\int_0^{20} (x+20)^2 dx = \int_{20}^{30} u^2 du = \int_{20}^{30} x^2 dx$$

(c) $u=\frac{1}{2}x,\ du=\frac{1}{2}dx$

$$\int_{2a}^{2b} \sqrt{\sin\frac{1}{2}x}\, dx = 2\int_a^b \sqrt{\sin u}\, du = 2\int_a^b \sqrt{\sin x}\, dx$$

4. $u=\ln\ln x,\ dv=xdx,\ du=\frac{1}{\ln x}\cdot\frac{1}{x}dx,\ v=\frac{1}{2}x^2$

$$\int_2^3 \ln\ln x dx = \frac{1}{2}x^2\ln\ln x\Big|_2^3 - \frac{1}{2}\int_2^3 \frac{xdx}{\ln x} = \frac{9}{2}\ln\ln 3 - 2\ln\ln 2 - \frac{1}{2}k$$

《복습문제》

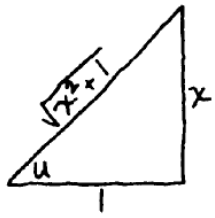
1. (a) 적분표 2와 비교하면 $a=1, b=0, c=1$

$\frac{1}{2}\ln(x^2+1)+C$

(b) $u=x^2+1, du=2xdx$라고 하자.

$\frac{1}{2}\int du/u=\frac{1}{2}\ln|u|+C=\frac{1}{2}\ln(x^2+1)+C$

(c) $\tan u=x, dx=\sec^2 u du, \sqrt{x^2+1}=\sec u$



$\int\frac{x}{x^2+1}dx=\int\frac{\tan u\sec^2 u du}{\sec^2 u}=\int\tan u du$

$=\ln|\sec u|+C$

$=\ln\sqrt{x^2+1}+C=\ln(x^2+1)^{1/2}+C$

$=\frac{1}{2}\ln(x^2+1)+C$

(d) $u=x, dv=\frac{1}{x^2+1}dx$라고 하자.

$du=dx, v=\tan^{-1}x,$

$\int\frac{x}{x^2+1}dx=xtan^{-1}x-\int\tan^{-1}xdx$

$=x\tan^{-1}x-(x\tan^{-1}x-\frac{1}{2}\ln(1+x^2))+C$

$=\frac{1}{2}\ln(x^2+1)+C$

2. (a) $u=x+2, du=dx$라고 하자.

$x=u=2$이므로,

$\int\frac{1}{u(u-2-4)}du=\int\frac{1}{u(u-6)}du$

$=-\frac{1}{-6}\ln\left|\frac{u-6}{u}\right|+C=\frac{1}{6}\ln\left|\frac{x-4}{x+2}\right|+C$

(b) $\int\frac{1}{x^2-2x-8}dx=\int\frac{1}{(x-1)^2-9}dx$

$=\frac{1}{6}\ln\left|\frac{x-1-3}{x-1+3}\right|+C=\frac{1}{6}\ln|\frac{x-4}{x+2}|+C$

(c) $\int\frac{1}{x^2-2x-8}dx=\frac{1}{6}\ln\left|\frac{2x-2-6}{2x-2+6}\right|+C=\frac{1}{6}\ln\left|\frac{x-4}{x+2}\right|+C$

(d) $\frac{1}{(x+2)(x+4)}=\frac{-1/6}{x+2}+\frac{1/6}{x-4}$

$\int\frac{1}{(x+2)(x+4)}=\int\left(\frac{-1/6}{x+2}+\frac{1/6}{x-4}\right)=\frac{1}{6}\ln|\frac{x-4}{x+2}|+C$

3. (a)공식 64

(b)$\frac{1}{3}ln|3x+4|+C$

(c)공식 19

(d) $u = 1+2x^3$

(e) $u = 3x$

(f) $-\frac{1}{6}e^{-6x}+C$

(g) $\frac{1}{5}ln|x|+C$

(h) 긴 나눗셈

(i) $x+3\ln|x|+C$

(j) 긴 나눗셈

(k) $u = 3x+4$

(l) 완전제곱식

(m) 부분적분

(n) 공식 1(b)

4. (a) 공식 45에 의해 $a=5, b=3$이면
$\frac{1}{4}\sin 2x-\frac{1}{16}\sin 8x+C$

(b) $\frac{1}{2}\int(\cos 2x-\cos 8x)dx = \frac{1}{4}\sin 2x-\frac{1}{16}\sin 8x+C$

(c) $u=\sin 3x, dv=\sin 5x dx, du=3\cos 3x dx, v=-\frac{1}{5}\cos 5x$
$\int \sin 3x \sin 5x dx = -\frac{1}{5}\sin 3x\cos 5x+\frac{3}{5}\int\cos 3x\cos 5x dx$
$u=\cos 3x$라고 하면, $dv=\cos 5x\, dx, du=-3\sin 3x dx, v=\frac{1}{5}\sin 5x$
$\int \sin 3x\sin 5x dx = -\frac{1}{5}\sin 3x\cos 5x+\frac{3}{5}(\frac{1}{5}\cos 3x\sin 5x+\frac{3}{5}\int\sin 3x\sin 5x dx)$
$\frac{16}{25}\int\sin 3x\sin 5x dx = -\frac{1}{5}\sin 3x\cos 5x+\frac{3}{25}\cos 3x\sin 5x$
$\int\sin 3x\sin 5x dx = \frac{5}{16}\sin 3x\cos 5x+\frac{3}{16}\cos 3x\sin 5x+C$
$=\frac{1}{4}\sin 2x-\frac{1}{16}\sin 8x+C$

5. $u=\tan x, dv=e^x dx$라 하자.
$du=\sec^2 x dx, v=e^x,$
$\int_0^{\pi/3} e^x\tan x dx = e^x\tan x|_0^{\pi/3}-\int_0^{\pi/3}e^x\sec^2 x dx$
$=e^{\pi/3}\sqrt{3}-Q$

6. $u=2+x^2, du=2xdx$
$\int_0^1 x(2+x^2)^5 dx = \frac{1}{2}\int_2^3 u^5 du = \frac{1}{12}u^6|_2^3 = 665/12$

7. (a) $-\cos x + C$

(b) $\frac{1}{2}(x - \sin x \cos x) + C$

(c) $\int \sin^3 dx = \int \sin x(1-\cos^2 x)dx = \int \sin x dx - \int \sin x \cos^2 x dx$

$u = \cos x,\ du = -\sin x dx$

$\int \sin x \cos^2 x dx = \int (-u^2) du = -\frac{1}{3}u^3 = -\frac{1}{3}\cos^3 x$

$\therefore -\cos x + \frac{1}{3}cos^3 x + C$

(d) $u = \sin x,\ du = \cos x dx$

$\int u du = \frac{1}{2}u^2 + C = \frac{1}{2}\sin^2 x + C$

(e) $u = \sin x,\ du = \cos x dx,\ \int u^2 du = \frac{1}{3}u^3 + C = \frac{1}{3}\sin^3 x + C$

(f) $-\frac{1}{4}\sin x \cos^3 x + \frac{1}{4}\int \cos^2 x dx$

$= -\frac{1}{4}\sin x \cos^3 x + \frac{1}{8}(x + \sin x \cos x) + C$

(g) $-1/x + C$

(h) $\ln|x| + C$

(i) $\frac{1}{2}\sqrt{x} + C$