

MSE, 미적분학

[연습문제 답안 이용 안내]

- 본 연습문제 답안의 저작권은 한빛아카데미(주)에 있습니다.
- 이 자료를 무단으로 전제하거나 배포할 경우 저작권법 136조에 의거하여 최고 5년 이하의 징역 또는 5천만원 이하의 벌금에 처할 수 있고 이를 병과(併科)할 수도 있습니다.

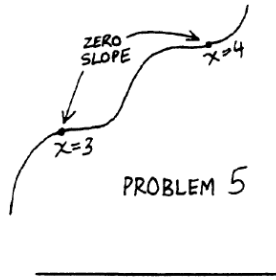
Chapter 04 연습문제 답안

《Section 4.1》

1.
 - (a) $f'(x) = 3x^2 - 6x - 24 = 3(x^2 - 2x - 8) = (x - 4)(x + 2) = 0$
 $x = 4, -2$
 극대 : $x = -2$, 극소 : $x = 4$
 - (b) $f'(x) = 4x^3 - 2x = 2x(2x^2 - 1) = 0$
 $x = 0, \pm \sqrt{\frac{1}{2}}$
 극대 : $x = 0$, 극소 : $x = \pm \sqrt{\frac{1}{2}}$
 - (c) $f''(x) = 5x^4 + 1$
 극대, 극소 없음.
 - (d) $f'(x) = (xe^x - e^x)/x^2$,
 $f' = 0$ or $xe^x - e^x = 0$,
 $e^x(x - 1) = 0, x = 1$
 극소 : $x = 1$
 - (e) $f'(x) = x \text{DOTS} \frac{1}{x} + \ln x = a + \ln x$
 $\ln x = -1, x = e^{-1}$
 극소 : $x = \frac{1}{e}$
2.
 - (a) 일차 미분 테스트 결과 $x = 2$ 에서 극소값을 갖는다.
 - (b) $x = 2$ 가 극대 또는 극소일 가능성이 있지만, 더 이상 결론을 내릴 수 없다.
 - (c) 어떤 결론도 내릴 수 없다.
 - (d) 극값이 존재하지 않는다.
 - (e) $x = 2$ 에서 극소값을 갖는다.
 - (f) $x = 2$ 가 극대 또는 극소일 가능성이 있지만, 더 이상 결론을 내릴 수 없다.
 - (g) $x = 7$ 가 극대 또는 극소일 가능성이 있지만, 더 이상 결론을 내릴 수 없다.
3. 아니요.
4.

x^3 은 $x = 0$ 에서 극값을 갖지 않는다.
 x^4 는 $x = 0$ 에서 극소값을 갖는다.
 $-x^4$ 는 $x = 0$ 에서 극대값을 갖는다.

5.



《Section 4.2》

1. (a) $f'(x) = 3x^2 + 2x - 5 = (3x + 5)(x - 1) = 0$.
 $x = -\frac{5}{3}, 1$
 (i) 최댓값 : ∞ , 최솟값 : $-\infty$
 (ii) 최댓값 : -3 , 최솟값 : $-\infty$
 (iii) 최댓값 : 0 , 최솟값 : -5
 - (b) $f'(x) = (xe^x - e^x)/x^2$
 $xe^x - e^x = 0$,
 (i) 최댓값 : ∞ , 최솟값 : $-\infty$
 (ii) 최댓값 : ∞ , 최솟값 : e
 (iii) 최댓값 : 0 , 최솟값 : $-\infty$
 - (c) $f'(x) = (-x^2 + 4x - 3)/(x^2 - 3)^2$
 $x = 3, 1$ 이면 $f' = 0$, $x = \pm\sqrt{3}$ 이면 $f' = \infty$
 (i) 최댓값 : ∞ , 최솟값 : $-\infty$
 (ii) 최댓값 : $\frac{1}{6}$, 최솟값 : e
 - (d) $f'(x) = 3x^2 + 2x - 1 = (3x - 1)(x + 1)$
 $f = -1, \frac{1}{3}$ 이면 $f' = 0$
 최댓값 : 79 , 최솟값 : $\frac{76}{27}$
2. 최댓값 : $f(3)$, 최솟값 : $f(4)$
 3. 최댓값 : ∞ , 최솟값 : $\sqrt{2}$
 4. $f(x) = x(300 - (x - 200)) = 500 - x^2$ ($200 \leq x \leq 350$)
 $x = 250$ 이면 $f' = 0$
 최대 수익을 위한 승객 : 250 명
 가장 작은 수익 : $62,500$ 원
 5. $\overline{CD} = x, \overline{AB} = 100 + x, \overline{BC} = \frac{1}{2}(200 - (100 + 2x)) = 50 - x$
 $A(x) = (100 + x)(50 - x) = 5000 - 50x - x^2$ ($0 \leq x \leq 50$)
 $A'(x) = -50 - 2x$
 $x = -25$ 이면 $A' = 0$ 이지만 범위 값이 아니다. $A(0) = 5000$

최댓값 : 5000, 최솟값 : 0

$$6. \quad x = \overline{BC}, \overline{AB} = 150 - x, \frac{\overline{EB}}{150 - x} = \frac{100}{150}, \overline{EB} = -\frac{2}{3}x + 100$$

$$A(x) = x\left(-\frac{2}{3}x + 100\right) = -\frac{2}{3}x^2 + 100x \quad (0 \leq x \leq 150)$$

$$A'(x) = -\frac{4}{3}x + 100$$

$x = 75$ 면 $A' = 0$ 이므로, 범위의 양 끝값과 75를 대입해보면 최대값은 $\overline{EB} = 50$

$$7. \quad p(x) = (100 + 1.2x)\left(12 - \frac{1}{40}x\right) = -0.03x^2 + 11.9x + 1200 \quad (0 \leq x \leq 480)$$

480일 이후 0으로 감소한다.

$$p'(x) = -0.06x + 11.9$$

$x = 11.9/0.06, 198.3$ 면 $p' = 0$ 이다.

$$p(198.3) = 23.8, p(0) = 12, p(480) = 0$$

즉 198일 후

$$8. \quad s(x) = -3x^2 - 10x - 13$$

$$s'(x) = f''(x) = -6x - 10$$

$x = -\frac{5}{3}$ 이 변곡점이지만 범위에 있는 값이 아니다.

$$s(0) = -13, s(1) = -26$$

가장 큰 기울기 : -13

가장 작은 기울기 : -26

$$9. \quad s(t) = \sqrt{(100 - 20t)^2 + 225t^2}, \quad 0 \leq t \leq 5$$

$$R(t) = (100 - 20t)^2 + 225t^2$$

$$R'(t) = -40(100 - 20t) + 450t$$

$$t = \frac{16}{5} \text{ 일 때 } R'(t) = 0$$

$$s\left(\frac{16}{5}\right) = 60, s(0) = 100, s(5) = 75$$

즉 $t = 3.2$, 3시간 12분 후

$$10. \quad \sqrt{(x-1)^2 + y^2} = \sqrt{(x-1)^2 + \frac{1}{9}(36 - 4x^2)} \quad (-3 \leq x \leq 3)$$

$$R(x) = (x-1)^2 + \frac{1}{9}(36 - 4x^2)$$

$$R'(x) = 2(x-1) - \frac{8}{9}x$$

$$x = \frac{9}{5} \text{ 일 때 } R'(x) = 0$$

$$A = (3, 0), B = (-3, 0), C = \left(\frac{9}{5}, \frac{8}{5}\right), D = \left(\frac{9}{5}, -\frac{8}{5}\right),$$

$$\overline{AQ} = 2, \overline{BQ} = 4, \overline{CQ} = \frac{1}{5} \sqrt{80}, \overline{DQ} = \frac{1}{5} \sqrt{80}$$

가장 가까운 점 : C, D , 가장 먼 점 : B

$$\begin{aligned} 11. \quad A(x) &= 2x\sqrt{r^2 - x^2} \quad (0 \leq x \leq r) \\ A'(x) &= 2x \frac{1}{2}(r^2 - x^2)^{-1/2} S - 2x + 2\sqrt{r^2 - x^2} \\ &= 2\sqrt{r^2 - x^2} - 2x^2 / \sqrt{r^2 - x^2} \end{aligned}$$

$$x = \pm r / \sqrt{2} \text{ 일 때 } A'(x) = 0$$

$$\text{최대 면적 : } \frac{2r}{\sqrt{2}}$$

$$12. \quad C = 600\left(5 + \frac{1}{10}s\right) + 600S \quad (30 \leq s \leq 80)$$

$$C'(s) = 60 + 360S(-600/s^2)$$

$s = \pm 60$ 일 때 $C'(s) = 0$ 이지만 -60 은 범위 밖이므로 무시

즉 후보는 $30, 60, 80$ 이고 $\$120, \$102, \$105$

가장 경제적인 속도 : 60 km/h

가장 경제적이지 않은 속도 : 30 km/h

$$\begin{aligned} 13. \quad \pi(16-x)^2/4\pi^2 &= (16-x)^2/4\pi \\ A(x) &= x^2/16 + (16-x)^2/4\pi \quad (0 \leq x \leq 16) \\ A'(x) &= \frac{x}{8} + \frac{1}{4\pi} \cdot 2(16-x) \cdot -1 = \frac{x}{8} - \frac{16-x}{2\pi} \end{aligned}$$

$$x = 64/(\pi+4) \text{ 일 때 } A'(x) = 0$$

$$A(64/(\pi+4)) = 64(\pi+4), A(0) = 64/\pi, A(16) = 16$$

모든 철사를 원 모양을 만드는 데에만 사용한다.

$$\begin{aligned} 14. \quad p &= 2A/x + 2x \quad (x \geq 0) \\ p'(x) &= -2A/x^2 + 2 \end{aligned}$$

$$x = \sqrt{A} \text{ 일 때 } p'(x) = 0$$

$$\text{최솟값은 } p(\sqrt{A}) = 4\sqrt{A}$$

$$15. \quad I(x) = x\left(-\frac{1}{2}x + 125\right) = 125x - \frac{1}{2}x^2 \quad (50 \leq x \leq 250)$$

$$I'(x) = 125 - x$$

$$x = 50, 125, 250 \text{ 일 때 } \$5000, \$7812.5, \$0$$

이므로, 최댓값은 125,000원

《Section 4.3》

1. (a) $0/0 = \lim_{x \rightarrow 1} (3x^2 - 5)/(2x - 3) = -2/-1 = 2$

(b) 2

(c) ∞

2. (a) ∞

(b) $0/0 = \lim \frac{1/(x-1)}{1} = 1$

(c) $-\infty/0+ = -\infty$

(d) 0

(e) $0/0 = \lim \frac{\cos x - 1}{-\sin x} = 0/0 = \lim \frac{-\sin x}{-\cos x} = \frac{0}{-1} = 0$

(f) $0/(1+0) = 0$

(g) $\frac{-\infty}{e^{\infty}} = \frac{-\infty}{\infty} = \lim \frac{1/x}{e^{1/x} \cdot -1/x^2} = \lim \frac{-x}{e^{1/x}} = \frac{0}{\infty} = 0$

(h) $-\infty/0+ = -\infty$

(i) 0

3. $\lim_{x \rightarrow \infty} \frac{(\ln x)^{27}}{x} = \frac{\infty}{\infty} = \lim \frac{27(\ln x)^{26} \cdot \frac{1}{x}}{1} = 27 \lim \frac{(\ln x)^{26}}{x} = \frac{\infty}{\infty}$

$27! \lim \frac{\ln x}{x} = 27! \lim \frac{1/x}{1} = 0$

즉 x 는 $(\ln x)^{27}$ 보다 빠르다.

4. (a) $\lim \frac{3\cos 3x}{2} = \frac{3}{2}$

(b) $\lim \frac{2\sin x \cos x}{1} = 0$

5. $\lim_{x \rightarrow 1} \frac{8x-2}{6x-4} = 3$

6. (a) 동일

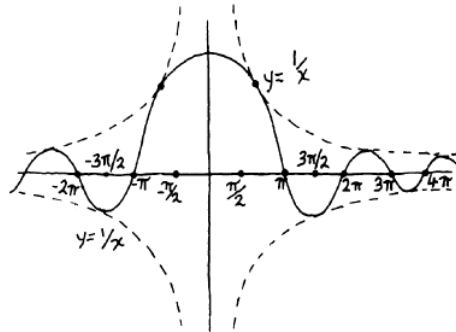
(b) $\lim_{x \rightarrow \infty} (e^{5x}/e^{3x}) = \lim 5e^{5x}/3e^{3x} = \infty/\infty e^{5x}$

$\lim_{x \rightarrow \infty} e^{5x}/e^{3x} = \lim e^{2x} = \infty$

즉 e^{5x}

(c) 동일

7.



《Section 4.4》

1. (a) $\infty \times e^{-\infty} = \infty \times 0$

$$\lim x/e^x = 0$$

(b) $0e^0 = 0 \times 1 = 0$

(c) $(-\infty)e^{\infty} = -\infty \times \infty = -\infty$

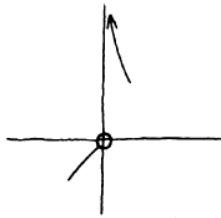
2. (a) $1 - \ln 1 = 1 - 0 = 1$

(b) $0 - (-\infty) = \infty$

(c) $\infty - \infty$

3. (a) $-\infty$ (b) $-\infty - 0 = -\infty$

4.



5. (a) 0

(b) $1 \times -\infty = -\infty$

(c) $\infty \times \sin 0 = \infty \times 0$
 $u = 1/x$

$$\lim_{u \rightarrow 0+} (\sin u)/u^2 = \frac{0}{0} = \lim_{u \rightarrow 0+} \cos u / 2u = 1/0+ = \infty$$

(d) $y = x^{1/x}$
 $\ln y = (1/x) \ln x$
 $\lim_{x \rightarrow \infty} \ln y = \lim_{x \rightarrow \infty} (\ln x)/x = 0, e^0 = 1$

(e) 0

(f) 1

(g) $\infty^{\infty} = \infty$

(h) $\infty \times (e^0 - 1) = \infty \times 0$

$$u = 1/x$$

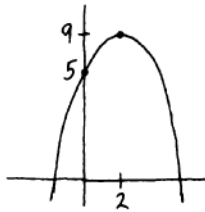
$$u \rightarrow 0+, \lim_{u \rightarrow 0+} (e^u - 1)/u = \lim_{u \rightarrow 0+} e^u / 1 = 1$$

(i) 0

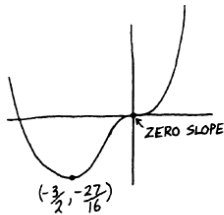
(j) e^{10}

《Section 4.5》

1.



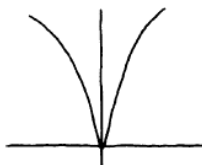
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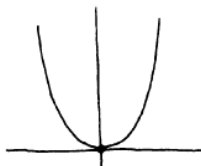
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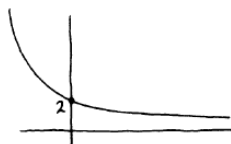
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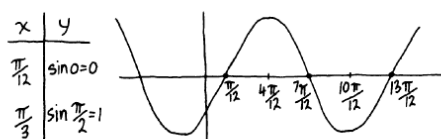
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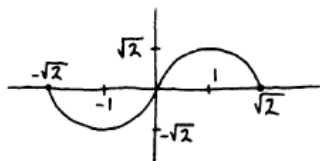
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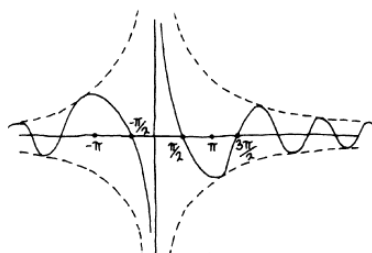
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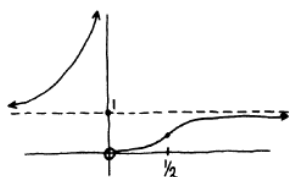
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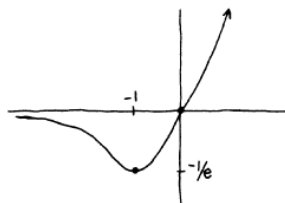
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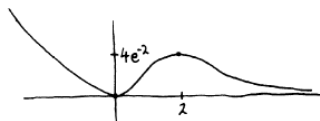
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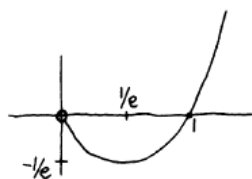
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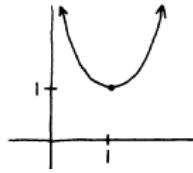
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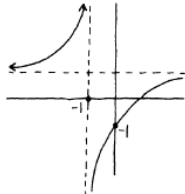
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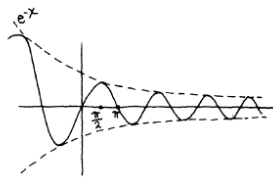
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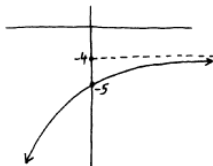
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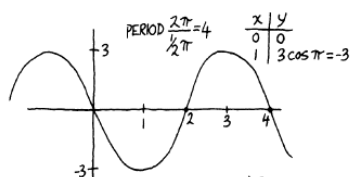
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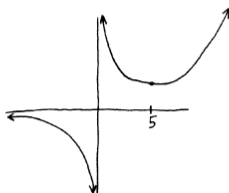
17.



18.



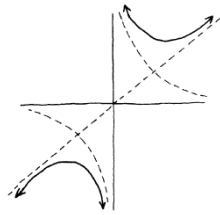
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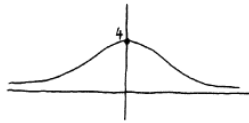
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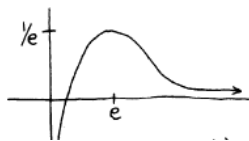
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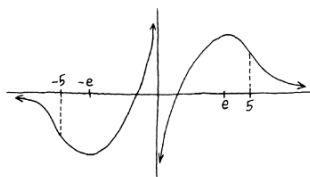
22.



23. (a)



(b)



《Section 4.6》

1. $V(t) = \frac{4}{3}\pi[r(t)]^3$

$$V'(t) = 4\pi r^2 r'$$

$$V' = -10$$

$$r = 2 \text{라고 하면 } -10 = 4\pi \cdot 4 \cdot r'$$

$$r' = -\frac{5}{8}\pi$$

$$-\frac{5}{8}\pi$$

2. $A(t) = b(t)h(t)$

$$A'(t) = b(t)h'(t) + h(t)b'(t)$$

$$b = 6, h = 8, b' = 4, h' = 3 \text{ 이면 } A' = 14$$

$$14 \text{ cm}^2/\text{s}$$

3. $s^2 = 8100 + x^2$

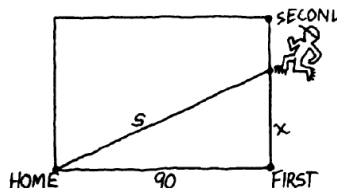
$$2ss' = 2xx'$$

$$s' = xx'/s$$

$$x = 30, x = 30\sqrt{10}, x(t) = 25$$

$$s' = (30)(25)/30\sqrt{10} = 25/\sqrt{10}$$

$$\frac{25}{\sqrt{10}} \text{ m}^2/\text{s}$$



4. $V(t) = \pi r^2 h(t) = 16\pi h(t)$

$$V'(t) = 16\pi h'(t)$$

$$h'(t) = V'(t)/16\pi = 8/16\pi$$

$$\frac{1}{2}\pi \text{ cm}^2/\text{min}$$

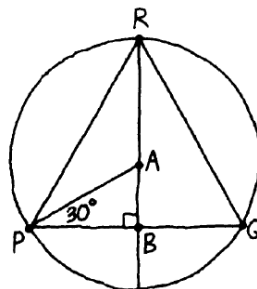
5. $\overline{AP} = r, \overline{AB} = \frac{1}{2}r, \overline{PB} = \frac{1}{2}r\sqrt{3}, \overline{RB} = r + \frac{1}{2}r = \frac{3}{2}r, \overline{PQ} = r\sqrt{3}$

$$A(t) = \frac{1}{2}bh = \frac{3}{4}\sqrt{3}[r(t)]^2$$

$$A'(t) = \frac{3}{2}\sqrt{3}rr' = \frac{9}{2}\sqrt{3}r \quad (r' = 3)$$

$$r = 4 \text{ 일 때 } A' = 18\sqrt{3}$$

$$18\sqrt{3} \text{ cm}^2/\text{s}$$



6. $x(t) = 5 \tan \theta(t), x'(t) = 5 \sec^2 \theta(t) \cdot \theta'(t) = 10 \pi \sec^2 \theta(t), \theta'(t) = 2\pi$

$x = 12$ 일 때 $\overline{BC} = 13, \sec \theta = \frac{13}{5}, x' = 338\pi/5$

$\frac{338\pi}{5} \text{ km/min}$

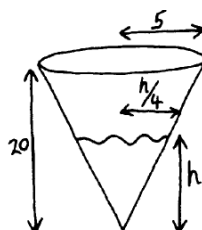
7. $V = \frac{1}{3} \pi r^2 h = \frac{1}{48} \pi h^3$

$V'(t) = \frac{1}{16} \pi h^2(t) h'(t)$

$h'(t) = 48/\pi h^2, V' = 3$

$h = 2$ 일 때 $h' = 12/\pi$

$\frac{12}{\pi} \text{ m/min}$



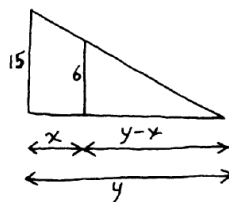
8. $\frac{y}{15} = \frac{y-x}{6}$

$6y = 15y - 15x$

$y = \frac{5}{3}x$

$y'(t) = \frac{5}{3}x'(t) = 5$

150 cm/sec



9. $V(t) = \frac{1}{3} \pi r^2 h = \frac{1}{12} \pi [h(t)]^3$

$V'(t) = \frac{1}{4} \pi h^2 h'$

(a) $V' = -10$

$h = 3$ 일 때 $h' = 4(-10)/9\pi = -40/9\pi \text{ cm/min}$

(b) $h = 6, h' = -2, V' = -18\pi$

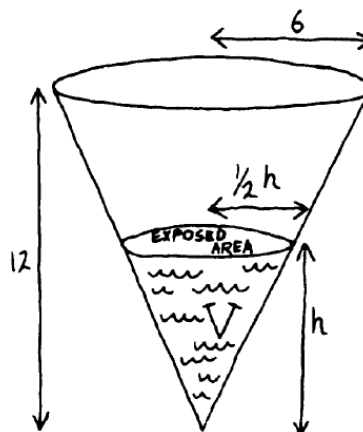
$18\pi \text{ cm/min}$

(c) $h = 2, r = 1, A = \pi r^2 = \pi$

$V' = -\sqrt{\pi}$

$h' = 4V'/\pi h^2 = -1/\sqrt{\pi}$

$\frac{1}{\sqrt{\pi}} \text{ cm/min}$



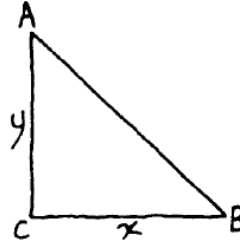
10. $A(t) = \pi [r(t)]^2, A'(t) = 2\pi r r' = 4\pi r$

$r = 5$ 일 때 $A' = 20\pi$

$20\pi \text{ m}^2/\text{sec}$

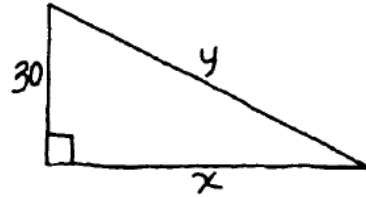
11. $A(t) = \pi[R(t)]^2 - \pi[r(t)]^2$
 $A'(t) = 2\pi R R' - 2\pi r r' = 4\pi R - 8\pi r$
 $r = 5, R = 9$ 일 때 $A' = 36\pi - 40\pi = -4\pi$
 $4\pi m^2/sec$

12. $A(t) = \frac{1}{2}x(t)y(t)$
 $A'(t) = \frac{1}{2}xy' + \frac{1}{2}x'y = 3x - 2y$
 $y = 12, x = 10$ 일 때 $A = 30 - 24 = 6$
 $6m^2/s$



13. $V(t) = \frac{4}{3}\pi[r(t)]^3$
 $V'(t) = 4\pi r^2 r' = -k(4\pi r^2)$
 $r' = V'/4\pi r^2 = -k(4\pi r^2)/4\pi r^2 = -k$
 $r' = -k$

14. $x^2(t) = y^2(t) - 900$
 $2xx' = 2yy'$
 $x = yy'/x = -2y$
 $y = 50$ 일 때 $x = 40, x' = -\frac{5}{2}$
 $y = 31$ 일 때 $x = \sqrt{61}, x' = -62/\sqrt{61}$
 $\frac{62}{\sqrt{61}}m/sec$



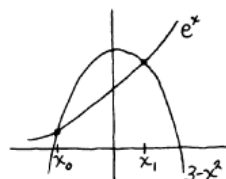
15. $R' = R^2(\frac{2}{R_1^2} + \frac{-3}{R_2^2})$
 $R_1 = 10, R_2 = 20$ 일 때
 $\frac{1}{R} = \frac{1}{10} + \frac{1}{20} = \frac{3}{20}$
 $R = \frac{20}{3}$
 $R' = \frac{400}{9}(\frac{2}{100} - \frac{3}{400}) = \frac{5}{9}$
 $\frac{5}{9}ohms/min$

《Section 4.7》

1. $f(x) = x^2 - 39$
 $f'(x) = 2x$,
 $new\ x = old\ x - \frac{(old)^2 - 39}{2 \times old}$
 $x = 6$ 이면 $new\ x = 6 - \frac{36 - 39}{12} = 6.25$
 $x = 6.25$ 이면 $new\ x = 6.25 - \frac{(6.25)^2 - 39}{12.5} = 6.245$
 $x = 6.245$ 이면 $new\ x = 6.244998$
 $f(6.24) < 0, f(6.245) > 0$ 이므로
 6.24는 정확한 숫자이다.

2. $x^3 = 173$
 $f(x) = x^3 - 173$
 $f'(x) = 3x, new\ x = old\ x - \frac{(old)^3 - 173}{3(old)^2}$
 5.5730028, 5.5720548, 5.5720549의 값이 나온다.
 정확도를 높이기 위해 값을 대입해보면
 $f(5.572054) < 0, f(5.5720548) > 0$ 이므로 소수점 아래 여섯자리까지는 정확하다.
 5.572054

3. $e^x = 3 - x^2$
 $f(x) = e^x + x^2 - 3$
 $f'(x) = e^x + 2x$
 $new\ x = old - \frac{e^{old} + (old)^2 - 3}{e^{old} + 2(old)}$
 가능한 근사값으로는 0.8477662, 0.8345815, 0.8344869가 나온다.
 정확도를 높이기 위해 값을 대입해보면
 $f(0.834) < 0, f(0.8345) > 0$ 이므로 소수점 아래 세 자리까지는 정확하다.



4. (a) 근이 없다. (b) 4.493

《Section 4.8》

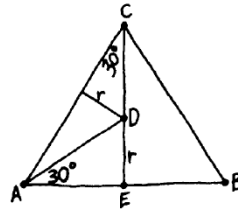
1. (a) $\frac{dx}{2\sqrt{x}}$ (b) $-\sin x dx$
 (c) $x^5 \cos x dx + 5x^4 \sin x dx$ (d) $\frac{x \cos x dx - \sin x dx}{x^2}$
 (e) $5x^4 \cos x^5 dx$ (f) 0

2. $6x^2 dx$

3. $df = dx$

4. (a) $d(x^3 + x^2) = (3x^2 + 2x)dx$
 $dx = -0.0001$ 으로부터 $d(x^3 + x^2) = -0.0033$
 (b) $d(x^{1/4}) = \frac{1}{4}x^{-3/4}$
 $d(x^{1/4}) = \frac{1}{4}(16)^{-3/4} \times 0.1 = (\frac{1}{4})(\frac{1}{8})(0.1) = \frac{1}{320}$

5. (a) $\overline{AE} = r\sqrt{3}$, $\overline{CD} = \overline{AD} = 2r$
 $A = \frac{1}{2}bh = \frac{1}{2}(2r\sqrt{3})(3r) = 3r^2\sqrt{3}$
 $dA = 6r\sqrt{3} dr$
 (b) $V = \frac{1}{3}\pi r^2 h$
 $dV = \frac{2}{3}\pi r h dr$



《Section 4.9》

1.
 - (a) $\cos y dy = -x dx$
 $\sin y = -\frac{1}{2}x^2 + C$
 - (b) $y dy = -dx/x^3$
 $\frac{1}{2}y^2 = \frac{1}{2x^2} + C$
 $y = \pm \sqrt{1/x^2 + D}$
 - (c) $y^4 dy = -x^2 dx$
 $\frac{1}{5}y^5 = -\frac{1}{3}x^3 + C$
 $y = \sqrt[5]{-\frac{5}{3}x^3 + D}$
 - (d) $dy/y = dx/(2x+3)$
 $\ln Ky = \frac{1}{2} \ln(2x+3) = \ln \sqrt{2x+3}$
 $Ky = \sqrt{2x+3}, y = A \sqrt{2x+3}$
 $y = A \sqrt{2x+3}$
 - (e) $e^{-y} dy = dx/x^2, -e^{-y} = -1/x + C$
 $e^{-y} = 1/x + D$
 $-y = \ln(1/x + D),$
 $y = -\ln(\frac{1}{x} + D)$
 - (f) $y dy = (5x+3)dx$
 $\frac{1}{2}y^2 = \frac{5}{2}x^2 + 3x + C$
 $y = \pm \sqrt{5x^2 + 6x + D}$
2.
 - (a) $dy/y = x dx,$
 $\ln Ky = \frac{1}{2}x^2, Ky = e^{x^2/2}, y = Ae^{x^2/2}$
 $3 = Ae^{1/2}, A = 3e^{-1/2}$
 $y = 3e^{(x^2-1)/2}$
 - (b) $y dy = (3-5x)dx$
 $\frac{1}{2}y^2 = 3x - \frac{5}{2}x^2 + C$
 $x=2, y=4 \Rightarrow C=12$
 $\frac{1}{2}y^2 = 3x - \frac{5}{2}x^2 + 12$
 $y = \sqrt{6x - 5x^2 + 24}$
 - (c) $e^y dy = 3x dx,$
 $e^y = \frac{3}{2}x^2 + C$
 $x=0, y=2 \Rightarrow C=e^2$

$$e^y = \frac{3}{2}x^2 + e^2$$

$$y = \ln\left(\frac{3}{2}x^2 + e^2\right)$$

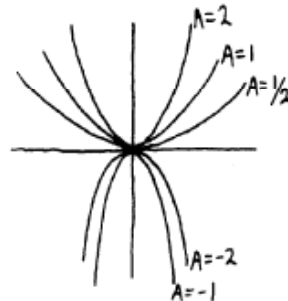
$$(d) \quad \frac{dy}{y^4} = \cos x dx$$

$$-1/3y^3 = \sin x + C$$

$$x = 0, y = 2 \Rightarrow \text{므로 } C = -1/24$$

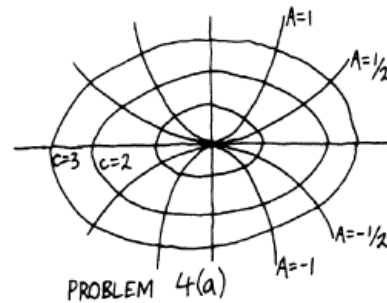
$$y = -\frac{1}{\sqrt[3]{3\sin x - \frac{1}{8}}}$$

3. (a) $dy/y = 2dx/x$
 $\ln Ky = 2\ln x = \ln x^2$
 $Ky = x^2$
 $y = Ax^2$

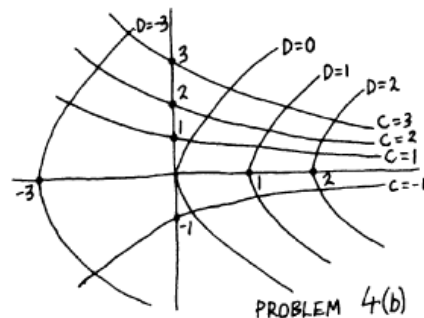


(b) $3 = 4A$
 $A = 3/4$
 $y = \frac{3}{4}x^2$

4. (a) $2x + 4yy' = 0$
 $y' = -2y/x, dy/y = -2dx/x$
 $\ln Ky = -2\ln x = \ln x^{-2}$
 $Ky = x^{-2}, y = Ax^{-2}$



(b) $ye^{3x} = C$
 $y3e^{3x} + y'e^{3x} = 0$
 $y' = -3y$
 $y' = \frac{1}{3y}, ydy = \frac{1}{3}dx$
 $\frac{1}{2}y^2 = \frac{1}{3}x + C$
 $x = \frac{3}{2}y^2 + D$



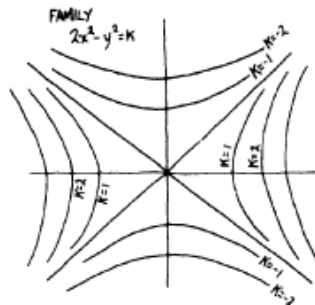
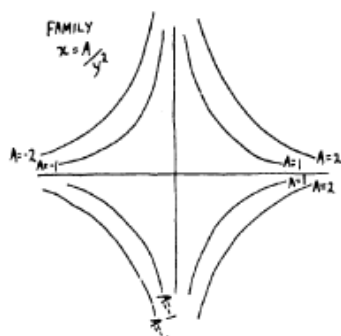
(c) $4x dx - 2y dy = 0$

$$dy/y = -dx/2x$$

$$\ln Ky = -\frac{1}{2} \ln x = \ln x^{-1/2}$$

$$Ky = \frac{1}{\sqrt{x}}, x = \frac{A}{y^2}, y = \pm x \sqrt{2}$$

$$\lim_{y \rightarrow 0} 1/y^2 = 1/0 = +\infty, \lim_{y \rightarrow \infty} 1/y^2 = 0, \lim_{y \rightarrow -\infty} 1/y^2 = 0$$



5. (a) $y'(t) = -\frac{1}{10}y(t)$

$$\frac{dy}{y} = -dt/10$$

$$\ln Ky = -\frac{1}{10}t$$

$$Ky = e^{-t/10}$$

$$y = Ae^{-t/10}$$

(b) $y = 75e^{-t/10}$

(c) $\frac{1}{2}A = Ae^{-t/10}$

$$\frac{1}{2} = e^{-t/10}$$

$$\ln \frac{1}{2} = -t/10$$

$$t = 10 \ln 2$$

6. $m'(t) = \frac{1}{2}m(t)$

$$dm/m = \frac{1}{2}dt$$

$$\ln Km = \frac{1}{2}t$$

$$Km = e^{t/2}$$

$$m = Ae^{t/2}$$

$$t = 3, m = 2e^{3/2}$$

7. $m \frac{dv}{dt} = mg - cv$

$$\frac{dv}{cv - mg} = - \frac{dt}{m}$$
$$\frac{1}{c} \ln K(cv - mg) = - \frac{t}{m}$$
$$K(cv - mg) = e^{-ct/m}$$
$$cv - mg = Ae^{-ct/m}$$
$$v = \frac{mg}{c} + \frac{A}{c} e^{-ct/m}$$
$$v(t) = \frac{mg}{c} (1 - e^{-ct/m})$$

《복습문제》

$$1. \quad \begin{aligned} PV &= kT \\ V &= kT/P \\ V'(t) &= k \left(\frac{PT' - TP'}{P^2} \right) \end{aligned}$$

$$T = 20, V = 10 \text{ 일 때 } P = 20k/10 = 2k, P' = 2, T' = -3$$

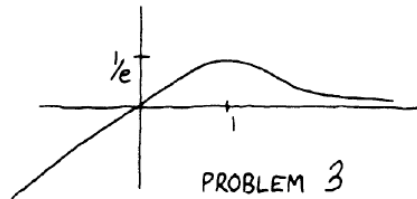
$$V' = k \left(\frac{6k + 40}{4k^2} \right) = \frac{3k + 20}{2k}$$

$$2. \quad (a) \quad \frac{\infty}{\infty} = \lim_{x \rightarrow \infty} \frac{\ln x}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{1}{\ln x} = \frac{1}{\infty} = 0$$

$$(b) \quad \ln 0^+ / \ln 1^+ = -\infty / 0^+ = -\infty$$

$$3. \quad f'(x) = e^{-x}(1-x) \text{ 는 } x=1 \text{ 일 때 } 0, x < 1 \text{ 일 때 양수, } x > 1 \text{ 일 때 음수}$$

$$f(-\infty) = -\infty \times \infty = -\infty$$



$$4. \quad \begin{aligned} p &= x(10-x) = -x^2 + 10x \\ p'(x) &= -2x + 10 \\ x = 5 \text{ 일 때 } p'(x) &= 0 \end{aligned}$$

$$5. \quad x d(e^{2x}) + e^{2x} d(x) = 2xe^{2x} dx + e^{2x} dx$$

$$6. \quad (a) \quad \ln x^2 = 2 \ln x, \text{ 동 일}$$

$$(b) \quad \lim_{x \rightarrow \infty} e^{x^2}/e^x = \infty/\infty = 2xe^{x^2}/e^x$$

$$\lim_{x \rightarrow \infty} e^{x^2}/e^x = \lim_{x \rightarrow \infty} e^{x^2-x} = e^{INF}$$

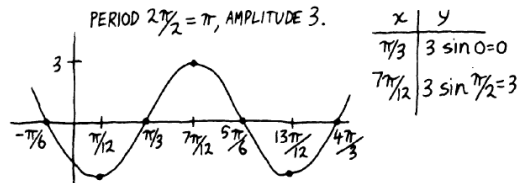
e^{x^2} 이 더 크다.

$$7. \quad \begin{aligned} V(t) &= [e(t)]^3 \\ V'(t) &= 3e^2 e' \end{aligned}$$

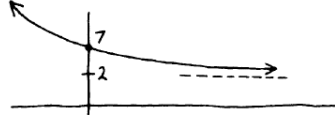
$$e = 3, e' = 2 \text{ 일 때 } V' = 54$$

$$54m^2/\text{sec}$$

8. (a)



(b)

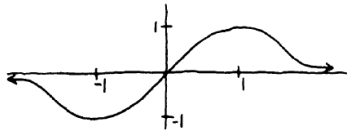


9. (a) $-\infty$

(b) 1

10. $A(x) = x \sqrt{d^2 - x^2}$
 $A'(x) = x \cdot \frac{1}{2}(d^2 - x^2)^{-1/2} \cdot -2x + \sqrt{d^2 - x^2} = (d^2 - 2x^2) / \sqrt{d^2 - x^2}$
 $x = d/\sqrt{2}$ 일 때 $\sqrt{d^2 - x^2} = \sqrt{d^2 - \frac{1}{2}d^2} = \frac{d}{\sqrt{2}}$

11.



12. (a) 극소 : $x = 0, \pi, \dots$ 극대 : $x = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$

(b) 극소 : $x = -2$

13. $dy/y = dt/t^2$
 $\ln Ky = -1/t$
 $Ky = e^{-1/t}$
 $y = 2e^{-1/t}$

14. $\overline{AD} = x, \overline{AB} = \frac{1}{2}(100 - 5x)$
 $A = x \frac{1}{2}(100 - 5x) = 50x - \frac{5}{2}x^2$
 $A'(x) = 50 - 5x$
 10×25

15. 최댓값 : 0, 최솟값 : $-\ln 2$

16. (a) $f'(x) = 3x^2 - 4x + 3 > 0$ (b) $f(x)$ 와 x 축과의 교차점은 1개이다.
- (c) $x = 1.5$ (d) 1.650