

10장 연습문제

1.

- (a) $\{t : t > 0, t \neq 1 \text{ 인 모든 실수}\}$
 (b) $\{t : t > 1\}$
 (c) $\{t : t > -2, t \neq -1, t \neq 1 \text{ 인 모든 실수}\}$
 (d) $\{t : t \geq 1\}$

3.

- (a) $\mathbf{r}'(t) = \langle 1, 2t, 3t^2 \rangle$, $\mathbf{r}''(t) = \langle 0, 2, 6t \rangle$
 (b) $\mathbf{r}'(t) = \langle \frac{1}{2}t^{-\frac{1}{2}}, \frac{1}{3}t^{-\frac{2}{3}}, \frac{1}{4}t^{-\frac{3}{4}} \rangle$, $\mathbf{r}''(t) = \langle -\frac{1}{4}t^{-\frac{3}{2}}, -\frac{2}{9}t^{-\frac{5}{3}}, -\frac{3}{16}t^{-\frac{7}{4}} \rangle$
 (c) $\mathbf{r}'(t) = \langle 2e^{2t}, 9(3t-1)^2, \frac{1}{t} \rangle$, $\mathbf{r}''(t) = \langle 4e^{2t}, 54(3t-1), -\frac{1}{t^2} \rangle$
 (d) $\mathbf{r}'(t) = \langle \sin t + t \cos t, te^t + e^t, 2t \rangle$, $\mathbf{r}''(t) = \langle 2\cos t - t \sin t, e^t(2+t), 2 \rangle$

5.

- (a) 속도벡터 $\mathbf{r}'(t) = \langle 3e^{3t}, 4t^3 - 2, -2\cos(3-2t) \rangle$
 (b) 속력 $|\mathbf{r}'(t)| = \sqrt{(3e^{3t})^2 + (4t^3 - 2)^2 + (-2\cos(3-2t))^2}$

$$= \sqrt{9e^{6t} + (4t^3 - 2)^2 + 4\cos^2(3-2t)}$$

 (c) 가속도벡터 $\mathbf{r}''(t) = \langle 9e^{3t}, 12t^2, -4\sin(3-2t) \rangle$

7.

- (a) $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|} = \frac{1}{2\sqrt{t^2+1}} \langle 2t, 2, 0 \rangle = \langle \frac{t}{\sqrt{t^2+1}}, \frac{1}{\sqrt{t^2+1}}, 0 \rangle$
 (b) $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|} = (1+t^2) \langle \frac{1}{(1+t^2)\sqrt{1+t^2}}, -\frac{t}{(1+t^2)\sqrt{1+t^2}}, 0 \rangle$

$$= \langle \frac{1}{\sqrt{1+t^2}}, -\frac{t}{\sqrt{1+t^2}}, 0 \rangle$$

9.

- (a) $\text{curl} \mathbf{F} = \langle 0, 0, 0 \rangle$, $\text{div} \mathbf{F} = 1 + 2y + 3z^2$
 (b) $\text{curl} \mathbf{F} = \langle 0, 0, e^y - 2x \rangle$, $\text{div} \mathbf{F} = 2y + xe^y + 2$
 (c) $\text{curl} \mathbf{F} = \langle -e^z, -\frac{1}{x}, -2y \rangle$, $\text{div} \mathbf{F} = 3$
 (d) $\text{curl} \mathbf{F} = \langle 2x^2y, xy(1-2y), -xz \rangle$, $\text{div} \mathbf{F} = yz$

11.

$$(a) \int_{C_1} (x^2 + y) dx + x^2 dy = \int_0^1 (2x^2 + 2x^3) dx = \frac{7}{6}$$

$$(b) \int_{C_2} (x + y) dx + x^3 y dy = \int_{-2}^2 (3x^8 + x^3 + x) dx = \frac{1024}{3}$$

(c)

$$\int_{C_3} \frac{y}{x} ds = \int_1^2 \frac{t^2}{t} \sqrt{1 + 4t^2} dt = \int_1^2 t \sqrt{1 + t^2} dt = \int_5^{17} \sqrt{q} dq = \frac{1}{12} (17\sqrt{17} - 5\sqrt{5})$$

13.

$$(a) W = \int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \langle x, y \rangle \cdot \langle \cos t, -\sin t \rangle dt = 0$$

$$(b) W = \int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \langle y, x \rangle \cdot \langle \cos t, -\sin t \rangle dt = \frac{1}{2}$$

15.

$\text{div} \mathbf{F} = 0$ 이다. 따라서 임의의 점에서 이상유체는 흘러 들어가거나 흘러 나가지 않는다.